Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

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So here is what we know so far:

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$

$\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Series – diverges

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ P-Series – converges if p>1, diverges otherwise

$\sum_{n=0}^{\infty} (r)^n$ Geometric – converges if $|r| < 1$ to $\frac{a_1}{1-r}$ and diverges if $|r| \geq 1$

Basic Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n > 0$

1. If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges

2. If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

Limit Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n \geq 0$

If you know $\sum_{n=1}^{\infty} b_n, b_n \geq 0$

1. If $\sum_{n=1}^{\infty} b_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ (L is any finite number),then $\sum_{n=1}^{\infty} a_n$ converges

2. If $\sum_{n=1}^{\infty} b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

The Integral Test:

If f is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ *and* $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

The Root Test:

Let $\sum a_k$ be a series with nonnegative terms. Suppose $(a_k)^{1/k} \to \rho$, then

1. $\sum a_k$ converges if $\rho < 1$
2. $\sum a_k$ diverges if $\rho > 1$
3. The test is inconclusive if $\rho = 1$

The Ratio Test:

Let $\sum a_k$ be a series with positive terms. Suppose $\frac{a_{k+1}}{a_k} \to \lambda$, then

1. $\sum a_k$ converges if $\lambda < 1$
2. $\sum a_k$ diverges if $\lambda > 1$
3. The test is inconclusive if $\lambda = 1$

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1. Use the Ratio test to determine if the following are convergent or divergent (or if test is inconclusive).

$$\sum \frac{2^n}{n!}$$

$$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$$

$$\lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$$

a. Converges   b. Diverges   c. Inconclusive

2-6 State whether each series converges (C) or diverges (D):

2. $\sum_{n=0}^{\infty}\left(\frac{1-5^n}{6^n}\right) = \sum_{n=0}^{\infty}\left(\frac{1}{6}\right)^n - \sum_{n=0}^{\infty}\left(\frac{5}{6}\right)^n$

3. $\sum_{n=1}^{\infty}\frac{(n-1)!}{n!} = \sum \frac{(n-1)!}{n\cdot(n-1)!} = \sum \frac{1}{n}$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n+3}}{n^3}$

5. $\displaystyle\sum_{n=1}^{\infty}(-1)^n$

6. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n^5+4n+3}$

$$\sum \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

9.6 Absolute Convergence and Alternating Series

An **alternating series** is a series whose terms alternate in sign. For example:

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots=\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{2^{n-1}}\right)$$

Alternating Series Test:

If an alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}(a_n)=a_1-a_2+a_3-a_4+\cdots, \quad a_n>0$

satisfies (*i*) $a_n \geq a_{n+1}$ for all *n* (non increasing) AND (*ii*) $\lim_{n\to\infty} a_n = 0$ then the series is convergent.

Without $(-1)^n$ part

$\sum \frac{(-1)^n}{n}$ converges by AST

$\sum \frac{1}{n}$ diverges

$\sum \frac{(-1)^n}{n^3}$

Examples:

1. $\sum \frac{(-1)^n}{\sqrt{n^2-1}}$ ← alt.

$\frac{1}{\sqrt{n^2-1}} \to 0$

Conv. by AST

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n^3+1}}{n+3}$ div. by BDT $\frac{\sqrt{n^3+1}}{n+3} \not\to 0$

3. $\sum(-1)^n$ div. by BDT

If an alternating series converges, it can be classified as either absolutely convergent or conditionally convergent.

If the series $\sum |a_n|$ is convergent, then $\sum a_n$ is convergent. We say that $\sum a_n$ is **absolutely convergent**.

If the series $\sum a_n$ is convergent and the series $\sum |a_n|$ is divergent, we say that $\sum a_n$ is **conditionally convergent**.

Determine if the following are absolutely convergent, conditionally conv. or divergent.

Examples:

4. $\sum \frac{(-1)^n}{\sqrt{n^2-1}}$

① alt & $\frac{1}{\sqrt{n^2-1}} \to 0 \Rightarrow$ conv. by AST

② classify the convergence

abs: $\sum \left| \frac{(-1)^n}{\sqrt{n^2-1}} \right| = \sum \frac{1}{\sqrt{n^2-1}}$ div. $\to$ not abs. conv.

$\Rightarrow$ conditionally convergent

5. $\sum \frac{\sin(\frac{\pi}{2}k)}{k\sqrt{k}}$ $\rightarrow$ $1, 0, -1, 0, 1, 0, -1, .$

$< \sum \frac{(-1)^{k+1}}{k^{3/2}}$ $\leftarrow$ alt. & $\frac{1}{k^{3/2}} \to 0 \Rightarrow$ conv.

$\boxed{\text{abs conv}}$: $\sum \left| \frac{(-1)^{k+1}}{k^{3/2}} \right| = \sum \frac{1}{k^{3/2}} \Rightarrow$ conv. by Pseries

6. $\sum(-1)^n \frac{n}{\sqrt{n^2-1}}$

alt a $\frac{n}{\sqrt{n^2-1}} \to \underline{\underline{1}} \neq 0$ diverges BDT

7. $\sum(-1)^k\left(\sqrt{k+1}-\sqrt{k}\right) \Leftarrow$ conditionally convergent.

alt & $\dfrac{\left(\sqrt{k+1}-\sqrt{k}\right)}{1}\dfrac{\left(\sqrt{k+1}+\sqrt{k}\right)}{\sqrt{k+1}+\sqrt{k}}$

$$= \frac{k+1-k}{\sqrt{k+1}+\sqrt{k}} = \frac{1}{\sqrt{k+1}+\sqrt{k}} \to 0$$

$\Rightarrow$ conv. by AST

$$= \sum(-1)^k \cdot \frac{1}{\sqrt{k+1}+\sqrt{k}}$$

abs? $\underline{\underline{\text{No}}}$ $\sum \dfrac{1}{\sqrt{k+1}+\sqrt{k}}$ (div)

If a convergent alternating series satisfies the condition $0 < a_{n+1} < a_n$, then the **remainder** R_N involved in approximating the sum S by S_N **is less in magnitude than the first neglected (truncated) term.** That is, $|R_N| = |S - S_N| \le |a_{N+1}|$

abs. value of next term

Examples:

1. Approximate the sum of $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n!}\right)$ by its first six terms and find the error.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n!}\right) \approx 1 + \frac{-1}{2} + \frac{1}{6} + \frac{-1}{24} + \frac{1}{120} + \frac{-1}{720}$$

$$= 91/144$$

$$R_6 \le \left|\frac{1}{5040}\right|$$

2. Find the smallest integer n so that s_n will approximate $\sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+2}$ to within 0.01

$$R_n \le \left| a_{n+1} \right|$$

$$\frac{1}{(n+1)^2+2} \le \frac{1}{100}$$

$$100 \le (n+1)^2 + 2$$

$$98 \le (n+1)^2$$

$$n+1 = 10$$

$$\boxed{n=9}$$

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7. Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum \frac{\cos(\pi k)}{k\sqrt{k}}$$

a) converges absolutely
b) ~~cannot be determined~~
c) converges conditionally
d) diverges