# Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

## Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

**Popper 28**

**1.** The series $\sum \frac{(-1)^k \sqrt{k+2}}{\sqrt{4k^3+2k+1}}$ is

alt

a. Absolutely convergent

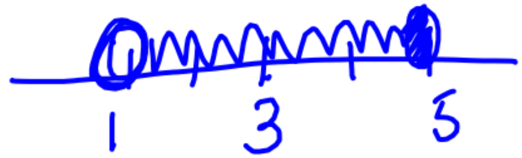
b. Conditionally convergent

~~c.~~ Divergent

?

$$\frac{\sqrt{k+2}}{\sqrt{4k^3+2k+1}} \to 0? \quad \text{yes}$$

$$\sum \frac{\sqrt{k+2}}{\sqrt{4k^3+2k+1}} \sim \sum \frac{1}{k}$$




Find the radius and interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n \cdot 2^n}$

look at $\displaystyle\sum \left| \frac{(-1)^n (x-3)^n}{n \cdot 2^n} \right| = \sum \frac{|x-3|^n}{n \cdot 2^n}$

Root: $\displaystyle\lim_{n\to\infty} \left( \frac{|x-3|^n}{n \cdot 2^n} \right)^{1/n} = \lim_{n\to\infty} \frac{|x-3|}{n^{1/n} \cdot 2}$ $\rightarrow 1$

$$= \frac{|x-3|}{2} < 1$$

$$|x-3| < 2 \Rightarrow R = 2$$

$x=1$: $\displaystyle\sum \frac{(-1)^n (-2)^n}{n \cdot 2^n} = \sum \frac{(-1)^n \cdot (-1)^n \cdot 2^n}{n \cdot 2^n} = \sum \frac{1}{n}$ div.

$x=5$: $\sum \frac{(-1)^n (2)^n}{n \cdot 2^n} = \sum \frac{(-1)^n}{n}$ conv. by AST

$(1, 5]$ int. of convergence

**Popper**

**2.** Find the radius of convergence for the power series $\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$

a) .5 b) 2 c) $\infty$ d) 0

**3.** Find the interval of convergence for the power series: $\sum_{n=0}^{\infty}\frac{1}{3^n}(x-1)^n$

a) (-2, 4) ~~b)~~ (-3, 3) c) $\left(\frac{2}{3},\frac{4}{3}\right)$ d) (2, 4)

$$\sum_{n=0}^{\infty}\left(\frac{x-1}{3}\right)^n$$

Find $\frac{d}{dx}\sum\frac{(-1)^k x^k}{3k^2+1}$

$$\frac{d}{dx} x^K = K x^{K-1}$$

$$= \sum \frac{(-1)^K K x^{K-1}}{3K^2+1}$$

**4.** Find $\dfrac{d}{dx}\sum \boxed{\dfrac{(-1)^k 2^k}{k^2+1}} x^k$

a. $\sum \dfrac{(-1)^k k 2^k}{k^2+1} x^{k-1}$

b. $\sum \dfrac{(-1)^k 2^k}{k(k^2+1)} x^{k-1}$

c. $\sum \dfrac{(-1)^k 2^k}{(k+1)(k^2+1)} x^{k+1}$

d. $\sum \dfrac{(-1)^k (k-1) 2^k}{k^2+1} x^{k-1}$

Find $\int \sum \frac{(-1)^k 3^k x^k}{k^2} dx$

$$\int x^k dx = \frac{x^{k+1}}{k+1} + C$$

$$(k \neq -1)$$

$$= \sum \frac{(-1)^k 3^k x^{k+1}}{k^2 (k+1)} + C$$

**5.** Find $\int \sum \frac{(-1)^k 2^k}{k^2+1} x^k \, dx$

a. $\sum \frac{(-1)^k k 2^k}{k^2+1} x^{k-1} + C$

b. $\sum \frac{(-1)^k 2^k}{k(k^2+1)} x^{k-1} + C$

c. $\sum \frac{(-1)^k 2^k}{(k+1)(k^2+1)} x^{k+1} + C$

d. $\sum \frac{(-1)^k (k-1) 2^k}{k^2+1} x^{k-1} + C$

Taylor Polynomials in x
Taylor Series in x

There are many functions that we only know at one point, or a handful of isolated points. Such as the trigonometric functions, $e^x$, *ln* x, etc.

$e^0 = 1$ $\ln 1 = 0$ $\ln e = 1$

Let's create a polynomial P(x) that has the same properties as some function $f$(x) that we know very well at x = a, such as *sin* (x) or $e^x$ around x = 0.

The properties that we need to consider are the function and derivative properties.

Why a polynomial?

nice derivatives + integrals
can plug in values + evaluate

$P_n(x) = a_0 + a_1 x^1 + a_2 x^2 + \cdots$ $\qquad a_n = \dfrac{f^k(c)}{k!}$

1) Find a polynomial of degree n = 4 for $f(x) = e^{2x}$ about x = 0.

centered $\qquad$ coeff. $\cdot (x-c)^k$

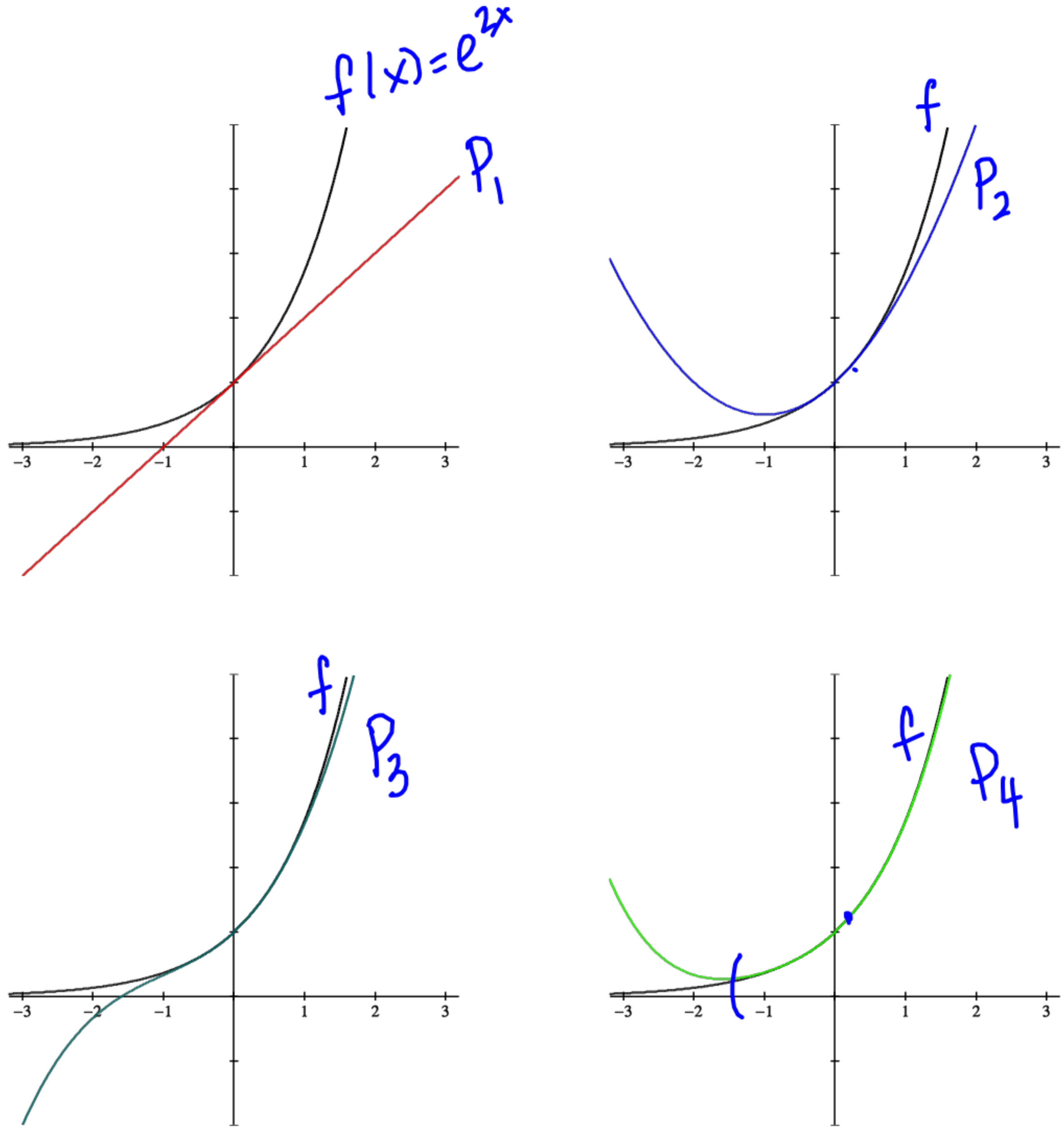
| k | $f^k(x)$ | $f^k(0)$ | $\dfrac{f^k(0)}{k!}$ | term |
|---|---|---|---|---|
| 0 | $e^{2x}$ | 1 | 1 | 1 |
| 1 | $2e^{2x}$ | 2 | 2 | $2x$ |
| 2 | $4e^{2x}$ | 4 | $\frac{4}{2!} = 2$ | $2x^2$ |
| 3 | $8e^{2x}$ | 8 | $\frac{8}{3!} = \frac{4}{3}$ | $\frac{4}{3}x^3$ |
| 4 | $16e^{2x}$ | 16 | $\frac{16}{4!} = \frac{2}{3}$ | $\frac{2}{3}x^4$ |

$P_4(x) = 1 + 2x + 2x^2 + 4/3\, x^3 + 2/3\, x^4$

$$f(x) = e^{2x}$$

$$P_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4$$

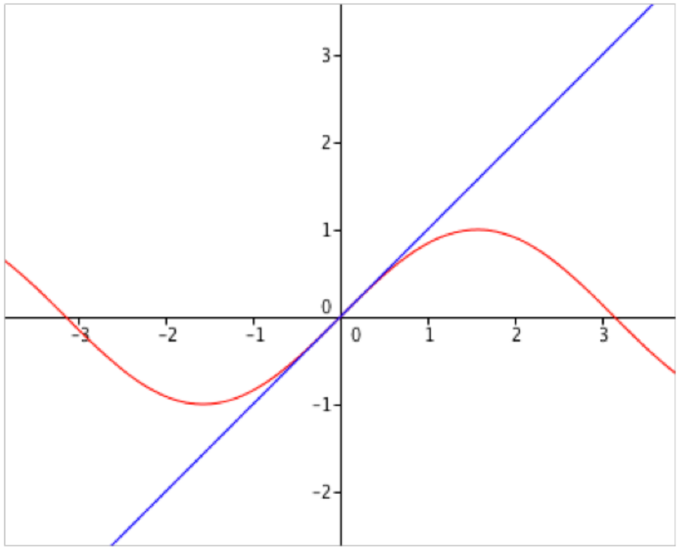
$$e^{1/2} = e^{2(1/4)} \approx P_4\left(\frac{1}{4}\right)$$


f(x)=e^{2x}
P_1
f
P_2
f
P_3
f
P_4
−3 −2 −1 1 2 3

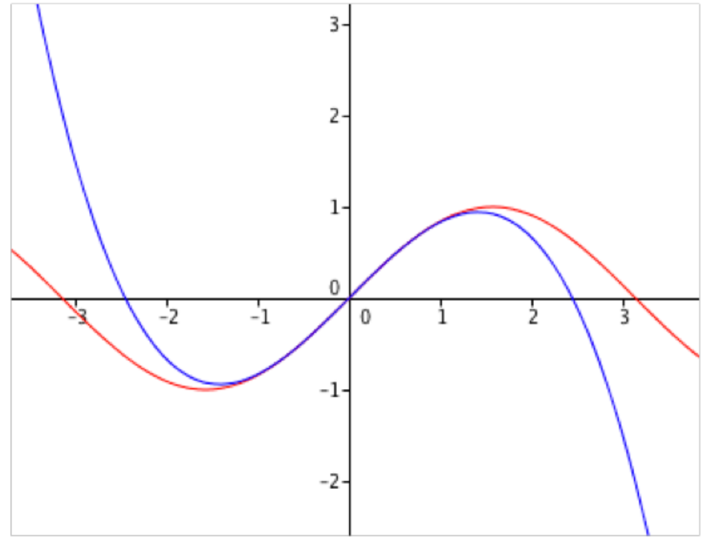
2) Find a polynomial of degree n = 5 for $f(x) = \sin x$ about x = 0.

| k | $f^k(x)$ | $f^k(0)$ | $\frac{f^k(0)}{k!}$ | term |
|---|---|---|---|---|
| 0 | sinx | 0 | 0 | — |
| 1 | cosx | 1 | 1 | x |
| 2 | -sinx | 0 | 0 | — |
| 3 | -cosx | -1 | $-1/3! = -1/6$ | $\frac{-1}{3!}x^3$ |
| 4 | sinx | 0 | 0 | — |
| 5 | cosx | 1 | $1/5! = 1/120$ | $\frac{1}{5!}x^5$ |
| | | | | |

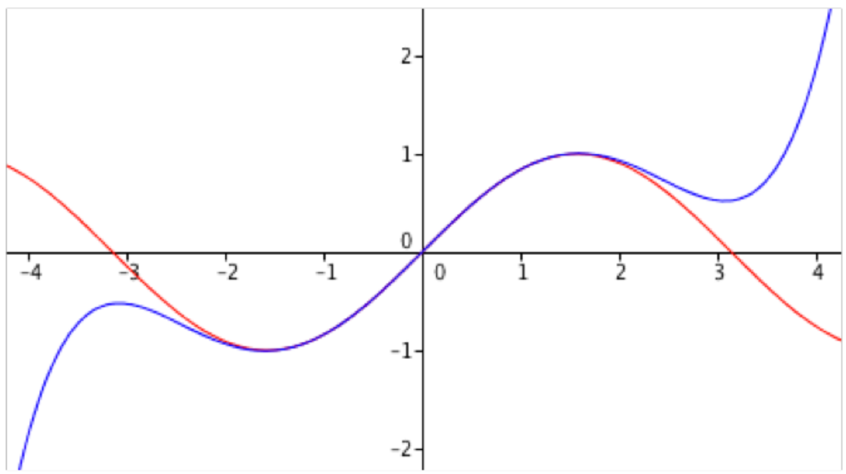
$$P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$



3
2
1
0
-1
-2
-3
-2
-1
0
1
2
3

3
2
1
0
-1
-2
-3
-2
-1
0
1
2
3

2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4

$$P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} = P_6(x)$$

$\cdot \sin(x^2)$

$\downarrow$

$$P_{10}(x) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!}$$

$$= x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!}$$

3) Find a polynomial of degree n = 4 for $f(x) = \ln|x+1|$ about $\underline{x = 0}$.

| k | $f^k(x)$ | $f^k(0)$ | $\frac{f^k(0)}{k!}$ | term |
|---|---|---|---|---|
| 0 | $\ln\|x+1\|$ | $\ln 1 = 0$ | 0 | — |
| 1 | $\frac{1}{x+1}$ | 1 | 1 | $x$ |
| 2 | $\frac{-1}{(x+1)^2}$ | $-1$ | $-\frac{1}{2}$ | $-\frac{1}{2}x^2$ |
| 3 | $\frac{2}{(x+1)^3}$ | 2 | $\frac{2}{3!} = \frac{1}{3}$ | $\frac{1}{3}x^3$ |
| 4 | $\frac{-6}{(x+1)^4}$ | $-6$ | $\frac{-6}{4!} = \frac{-1}{4}$ | $-\frac{1}{4}x^4$ |
| | | | | |

$$P_4(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

4) Use the Taylor approximation $e^x \approx \underbrace{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}}$ for x near 0 to find:

$\lim_{x \to 0} \frac{e^x - 1}{2x}$.

$$\lim_{x \to 0} \frac{\cancel{1} + x + \frac{x^2}{2!} + \frac{x^3}{3!} - \cancel{1}}{2x}$$

$$\lim_{x \to 0} \frac{1 + \frac{x}{2!} + \frac{x^2}{3!}}{2} = \frac{1}{2}$$

5) Use the Taylor approximation $\sin x \approx x - \dfrac{x^3}{3!}$ for x near 0 to find

$\lim_{x \to 0} \dfrac{\sin x}{x}$.

$$\lim_{x \to 0} \frac{x - x^3/3!}{x} = \lim_{x \to 0} 1 - x^2/3!$$

$$= 1$$

**Definition of nth degree Taylor polynomial:**

If $f$ has n derivatives at c, then the polynomial

value we will approximate

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the nth degree Taylor polynomial for $f$ at c.

centered

**If c = 0,** then

$$P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \ldots + \frac{f^{(n)}(0)}{n!}(x)^n$$

may be called the nth degree **Maclaurin** polynomial for $f$.

6) Give the $8^{th}$ degree Taylor polynomial approximation to ln ($x$) centered at $x = 1$.

$$P_8(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 \rightarrow$$

| k | $f^k(x)$ | $f^k(1)$ | $\frac{f^k(1)}{k!}$ | term |
|---|---|---|---|---|
| 0 | $\ln x$ | 0 | 0 | — |
| 1 | $\frac{1}{x}$ | 1 | 1 | $(x-1)$ |
| 2 | $-1/x^2$ | $-1$ | $-1/2$ | $-\frac{1}{2}(x-1)^2$ |
| 3 | $2/x^3$ | 2 | $2/3! = 1/3$ | $\frac{1}{3}(x-1)^3$ |
| 4 | $-6/x^4$ | $-6$ | $-6/4! = -1/4$ | $-\frac{1}{4}(x-1)^4$ |

$$-\frac{1}{6}(x-1)^6+\frac{1}{7}(x-1)^7-\frac{1}{8}(x-1)^8$$

Poppers 6&7 each C