Math 1432

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Office Hours:

Mondays 1-2pm,
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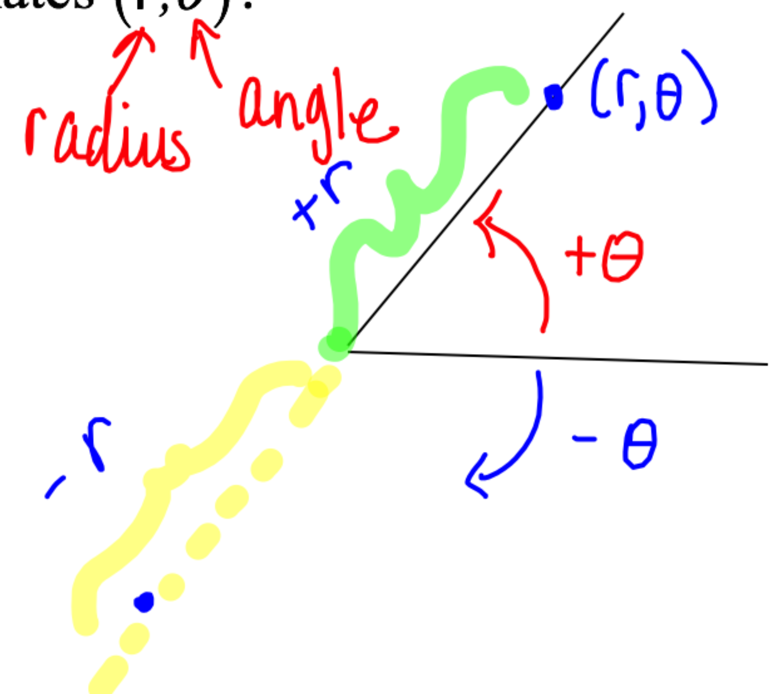
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

POPPER 31 ("Pop 31" under EMCF)

1. If $r \neq 0$, which of the following polar coordinate pairs represents the same point as the point with polar coordinates (r, θ)?



a. $(-r, \theta)$

b. $(-r, \theta + 2\pi)$

c. $(-r, \theta + 3\pi)$

d. $(r, \theta + \pi)$

e. $(r, \theta + 3\pi)$

ex: $\left(2, \frac{\pi}{3}\right)$

a: $\left(-2, \frac{\pi}{3}\right)$ b. $\left(-2, \frac{7\pi}{3}\right)$

2. Which of the following are the rectangular coordinates of the point with polar coordinate $\left(\underset{r}{-2}, \underset{\theta}{\frac{-\pi}{3}}\right)$?

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

a. $\left(-\sqrt{3}, 1\right)$

b. $\left(-1, \sqrt{3}\right)$

c. $\left(-\sqrt{3}, -1\right)$

d. $\left(-1, -\sqrt{3}\right)$

e. none of these

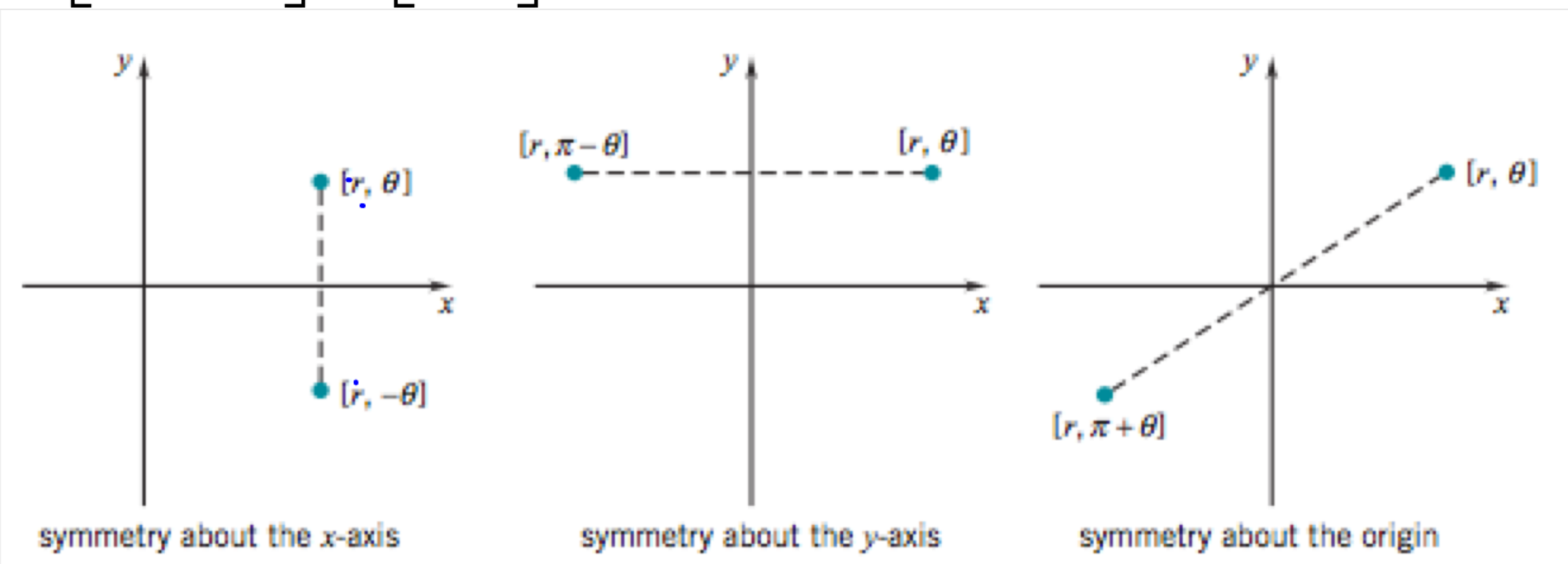
Graphing Polar Equations

Testing for Symmetry

If $[r, -\theta] \Rightarrow [r, \theta]$ then the graph is symmetric about the $x-$axis.

If $[r, \pi - \theta] \Rightarrow [r, \theta]$ then the graph is symmetric about the $y-$axis .

If $[r, \pi + \theta] \Rightarrow [r, \theta]$ then the graph is symmetric about the origin.



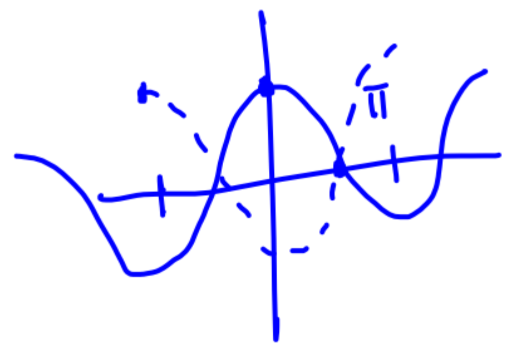
$r \quad \theta$

Find points of symmetry of $\left[2, \frac{1}{3}\pi\right]$ about:

a) x-axis $\left[2, -\pi/3\right]$

b) y-axis $\left[2, \pi - \pi/3\right] = \left[2, 2\pi/3\right]$

c) origin $\left[2, \pi + \pi/3\right] = \left[2, 4\pi/3\right]$



Test $r = 2 + \cos\theta$ for symmetry.

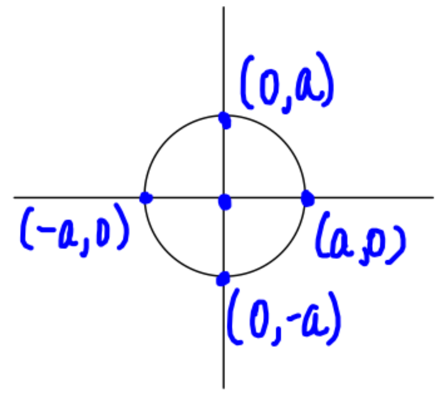
x-axis: $r = 2 + \cos(-\theta) = 2 + \cos(\theta)$ ✓

y-axis: $r = 2 + \cos(\overset{-\theta+\pi}{\pi - \theta}) = 2 - \cos\theta$

origin: $r = 2 + \cos(\pi + \theta) = 2 - \cos\theta$

Circles

Circle centered at (0, 0) with radius a.

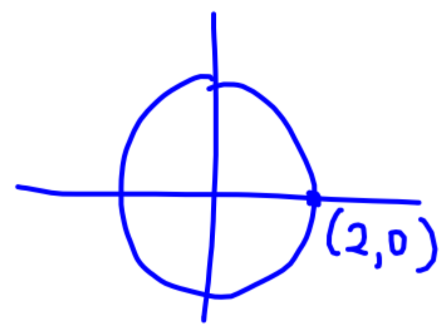


Cartesian: $x^2 + y^2 = a^2$

Polar: $r^2 = a^2$

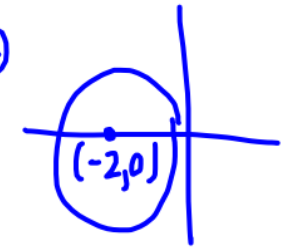
$\boxed{r = a}$

$r = 2$:

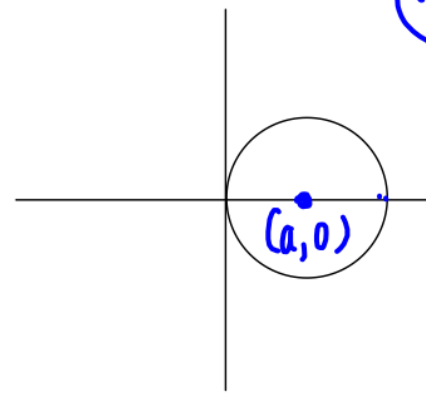


$$(x-h)^2+(y-k)^2=r^2$$

$$r=-4\cos\theta$$



Circle centered at (a, 0) with radius a.



Cartesian: $(x-a)^2+y^2=a^2$

$$x^2-2ax+\underline{a^2}+y^2=\underline{a^2}$$

Polar:

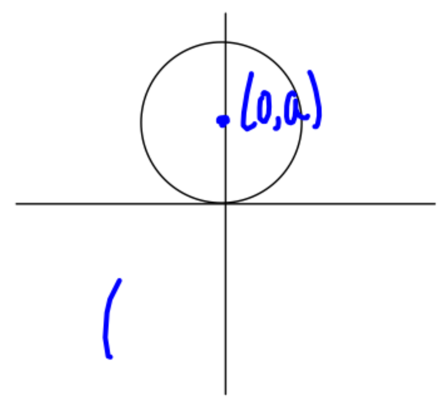
$$x^2+y^2=2ax$$

$$r^2=2ar\cos\theta$$

$$\boxed{r=2a\cos\theta}$$

$$\underset{\text{diam.}}{r=\underset{\uparrow}{3}\cos\theta}$$

Circle centered at (0, a) with radius a.



Cartesian: $x^2+(y-a)^2=a^2$

$$x^2+y^2=2ay$$

Polar:

$$r^2=2ar\sin\theta$$

$$\boxed{r=2a\sin\theta}$$

Lines

Horizontal Lines: $y = a$

$r\sin\theta = a$

$r = a/\sin\theta$ or $r = a\csc\theta$

Vertical Lines: $x = a$

$r\cos\theta = a$

$r = a/\cos\theta$ or $r = a\sec\theta$

Lines through the origin: $y = mx$

$r\sin\theta = mr\cos\theta$

$\sin\theta = m\cos\theta$

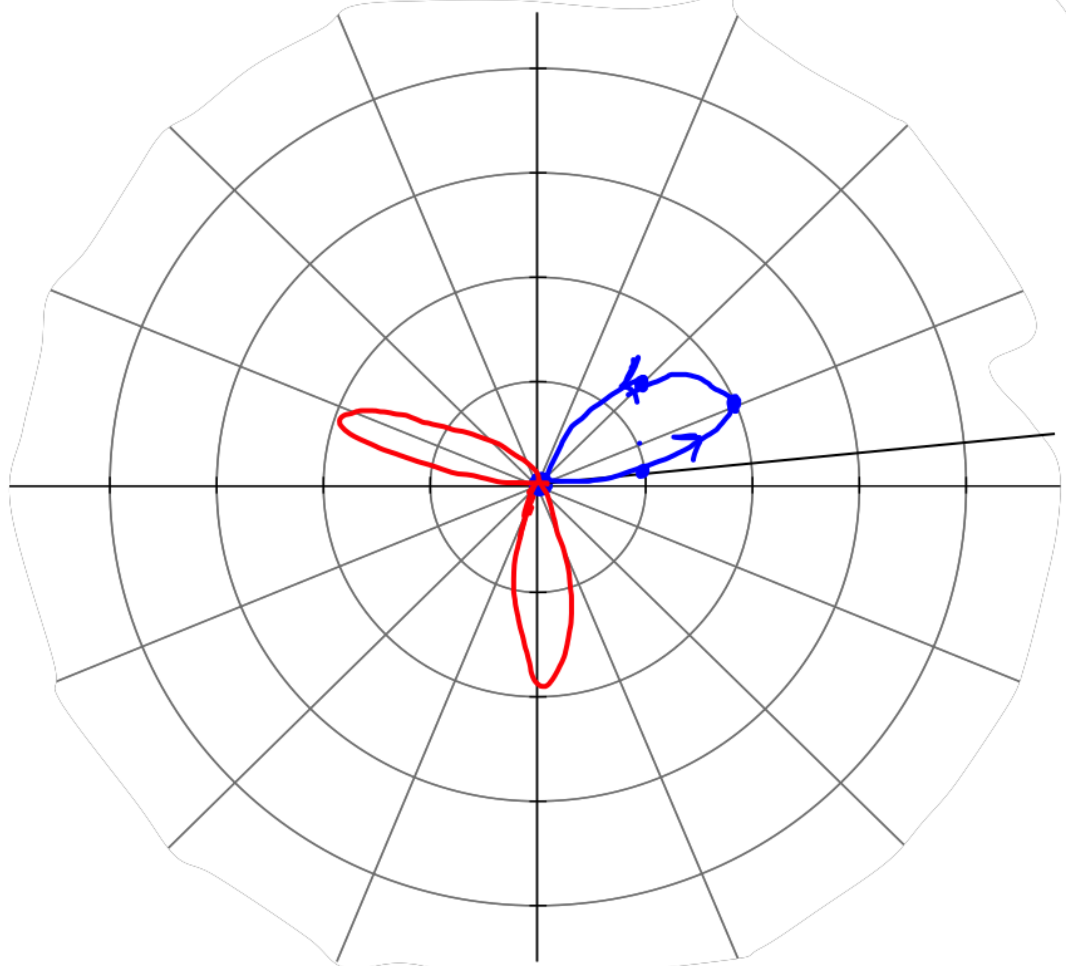
$\tan\theta = m$ / $\theta = \tan^{-1} m$

Arbitrary Lines: $y = mx + b$

$r\sin\theta = mr\cos\theta + b$

Sketch a graph of r = 2*sin* (3θ)

θ	r
0	$2\sin(0)=0$
$\pi/18$	$2\sin(\pi/6)=1$
$\pi/6$	$2\sin(\pi/2)=2$
$\pi/4$	$2\sin\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{}$
$\pi/3$	$2\sin(\pi)=0$



Polar graphs that produce flowers

length

$$\begin{cases} r = a\cos(m\theta) \\ r = a\sin(m\theta) \end{cases}$$

always have petal on pos. x-axis

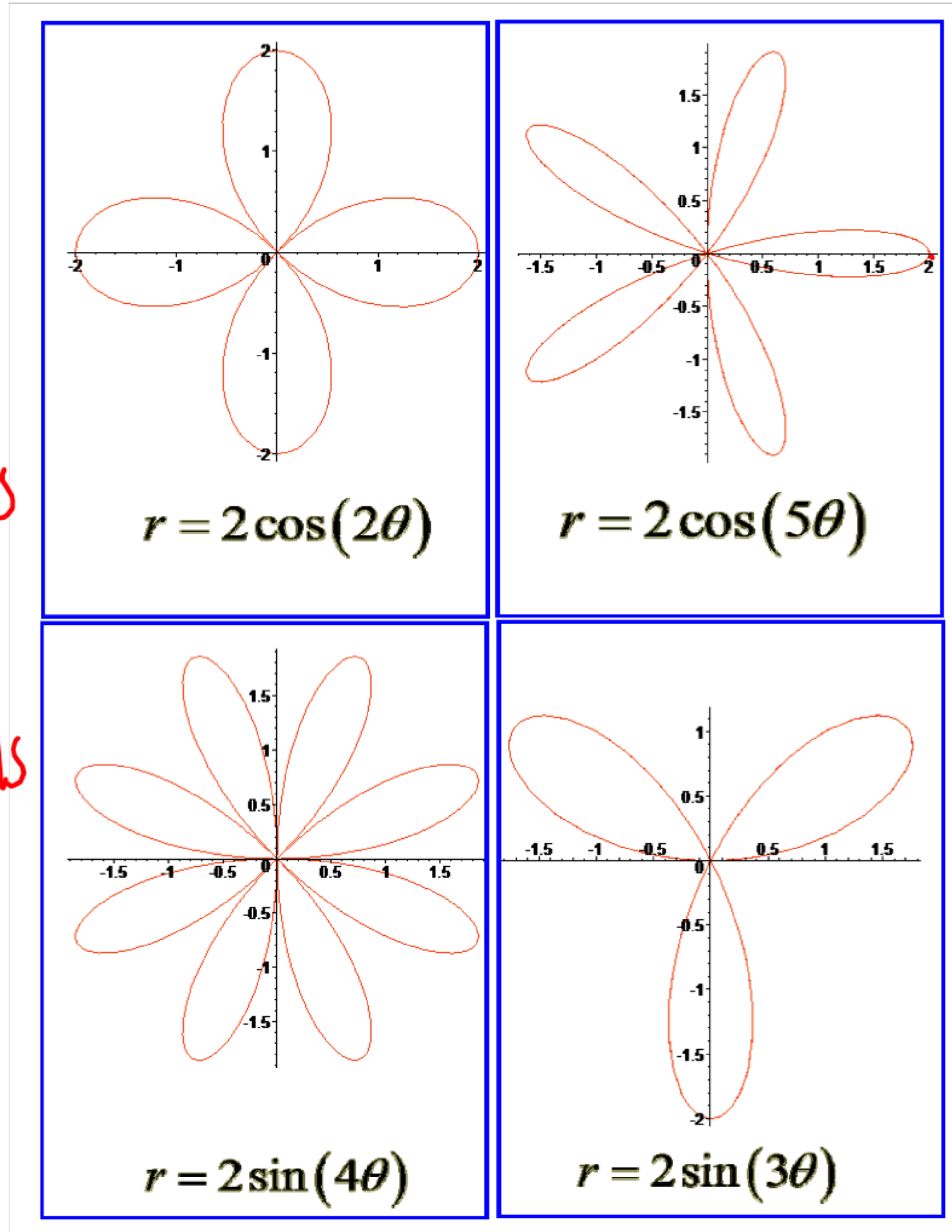
doesn't.

a > 0 and m is a positive integer

$m \neq 1$

m even
use 2m petals

m odd
use m petals



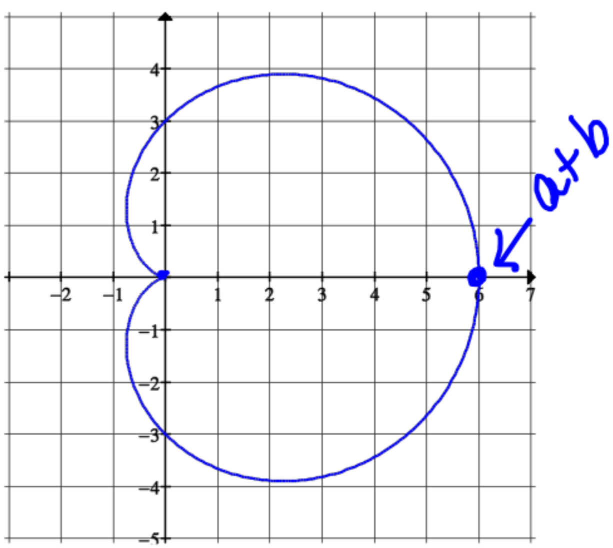
Polar Curves of the form

$r = a \pm b\, cos(\theta)$

and

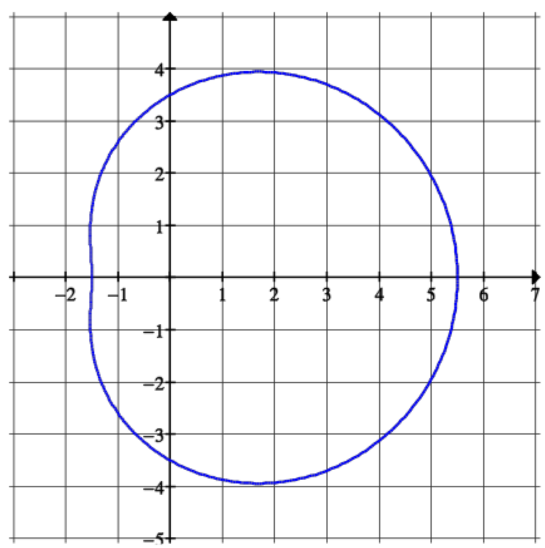
$r = a \pm b\, sin(\theta)$

Cardiods, Limaçons with dimples and Limaçons with inner loops



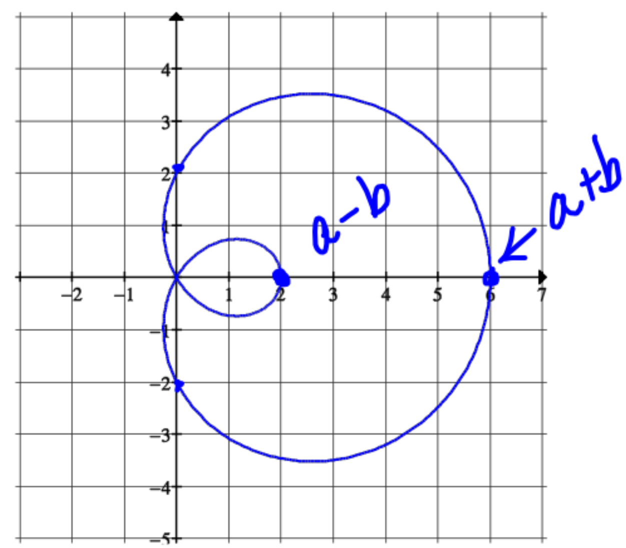
Cardioid

$|a| = |b|$



Limaçon with dimple

$|a| > |b|$

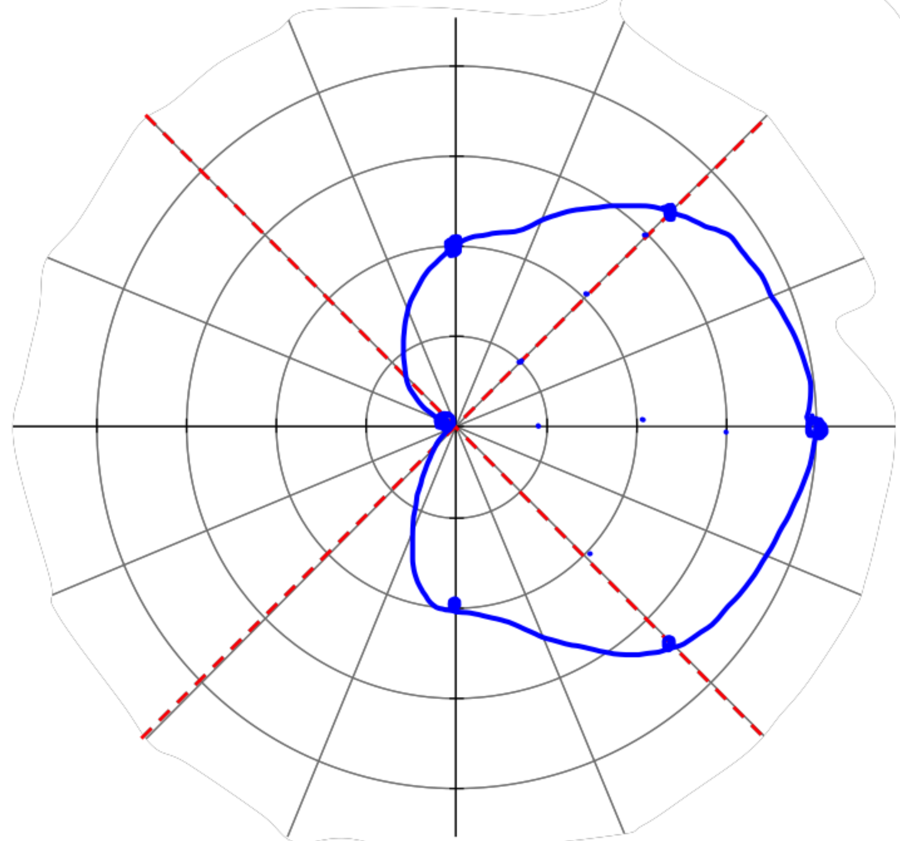


Limaçon with loop

$|a| < |b|$

Graph: $r = 2 + 2\cos\theta$

θ	r
0	$2+2(1)=4$
$\pi/4$	$2+2\left(\frac{\sqrt{2}}{2}\right)=3.4$
$\pi/2$	$2+0$
π	$2-2=0$

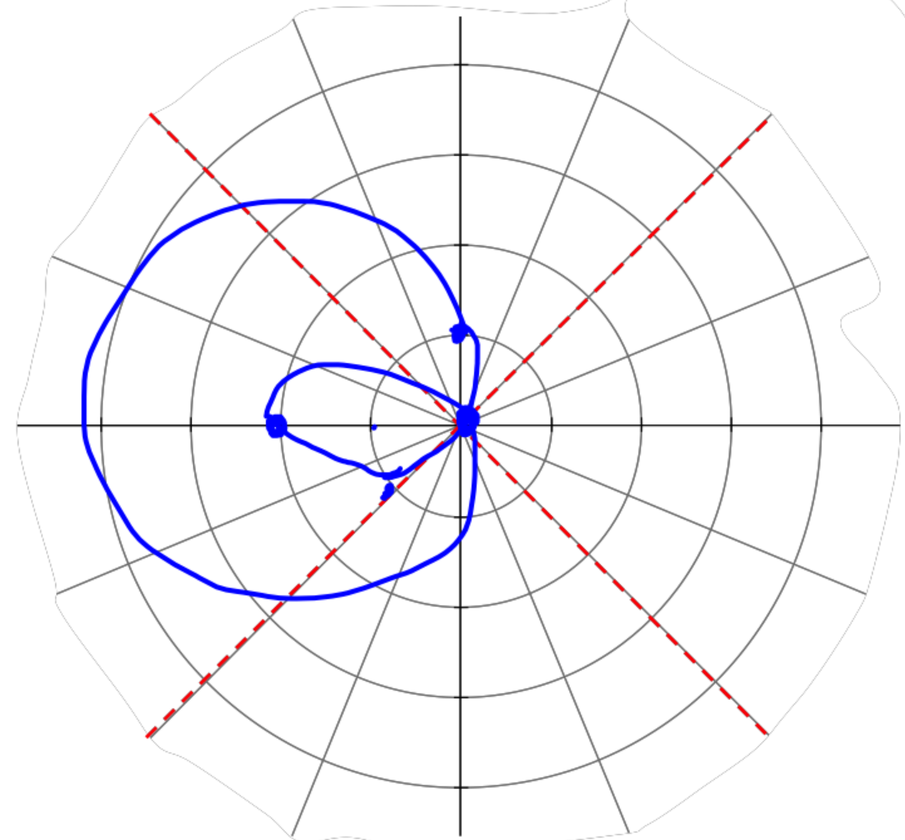


Graph: $r = 1 - 3\cos\theta$

Loop

$|a| < |b|$

θ	r
0	$1-3(1) = -2$
$\pi/4$	$1-3(\sqrt{2}/2)$
$\pi/2$	1

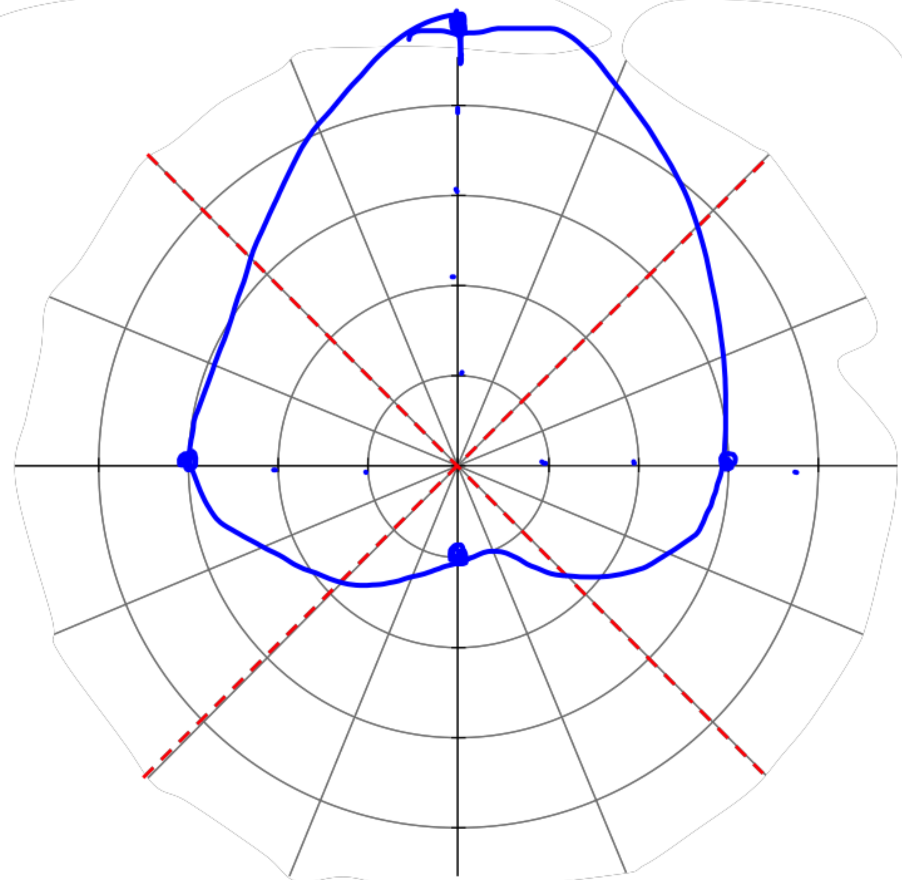


$0 = 1 - 3\cos\theta$

Graph: $r = 3 + 2\sin\theta$

$|b| > |a|$

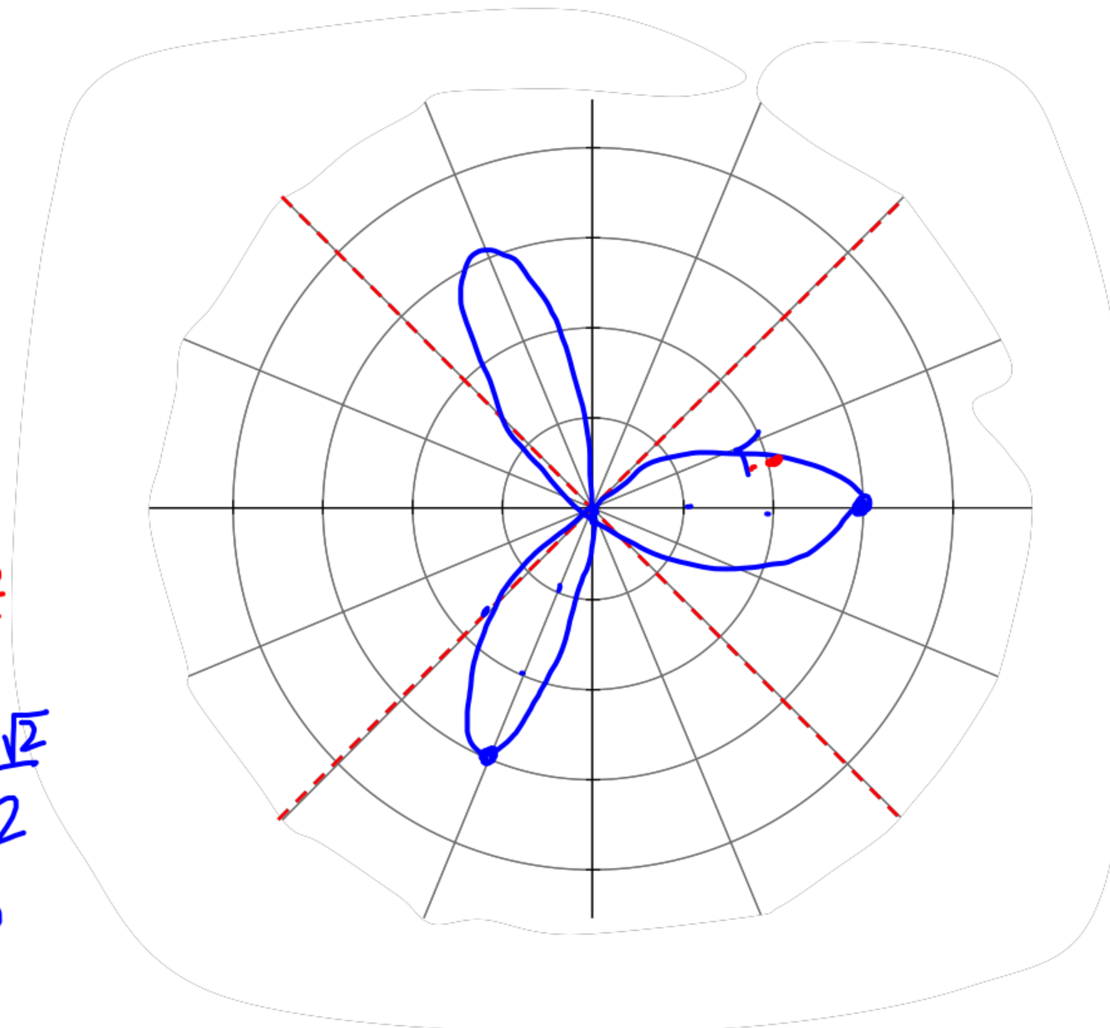
dent



Graph: $r = 3\cos 3\theta$

3 petals

θ	r
0	3
$\pi/6$	$3\cos(\pi/2) = 0$
$\pi/12$	$3\cos(\pi/4) = \frac{3\sqrt{2}}{2}$
$\pi/4$	$3\cos(3\pi/4) = \frac{3\sqrt{2}}{2}$
$\pi/3$	$3\cos(\pi) = -3$



POPPER 31

3. $\sum \frac{1}{n}$

a. converges
b. diverges

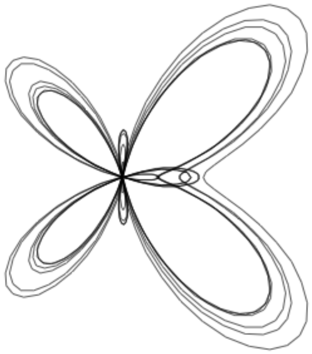
4. $\sum \frac{1}{n^2}$

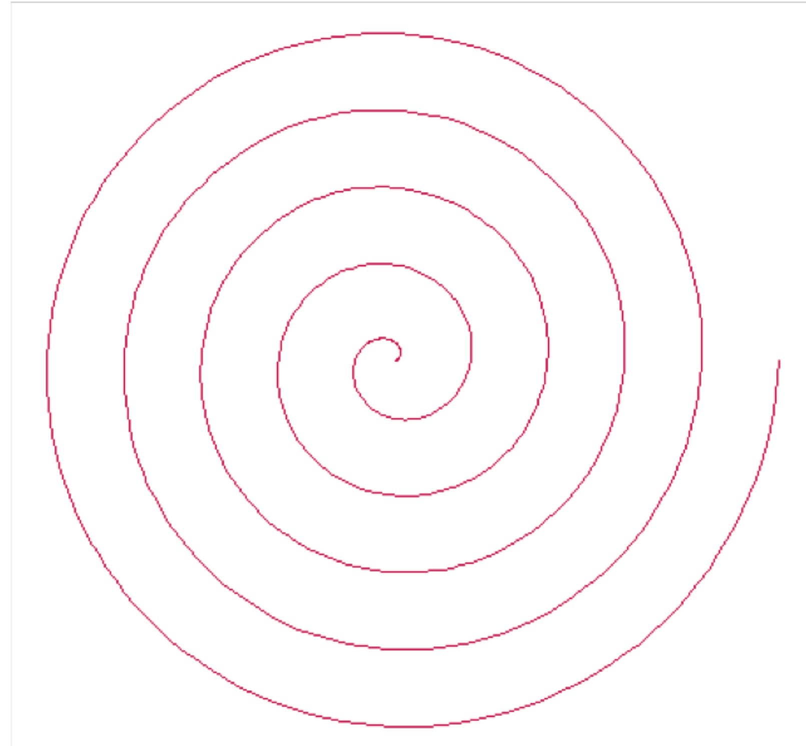
a. converges
b. diverges

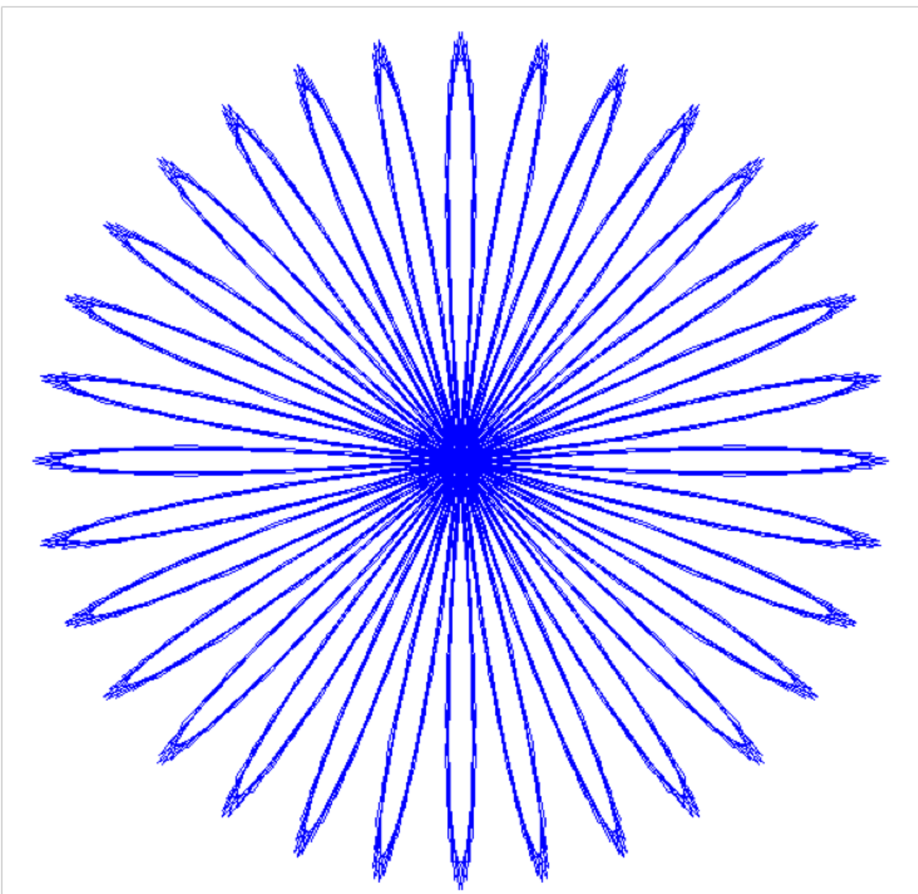
5. $\sum \frac{(-1)^n}{n}$

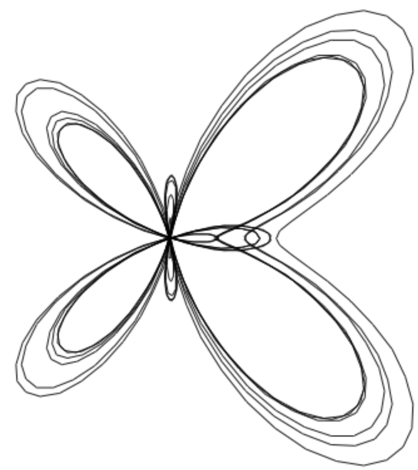
a. converges
b. diverges

Cool polar graphs made with "Winplot"









Graph: $r = 2 + 2\sin\theta$

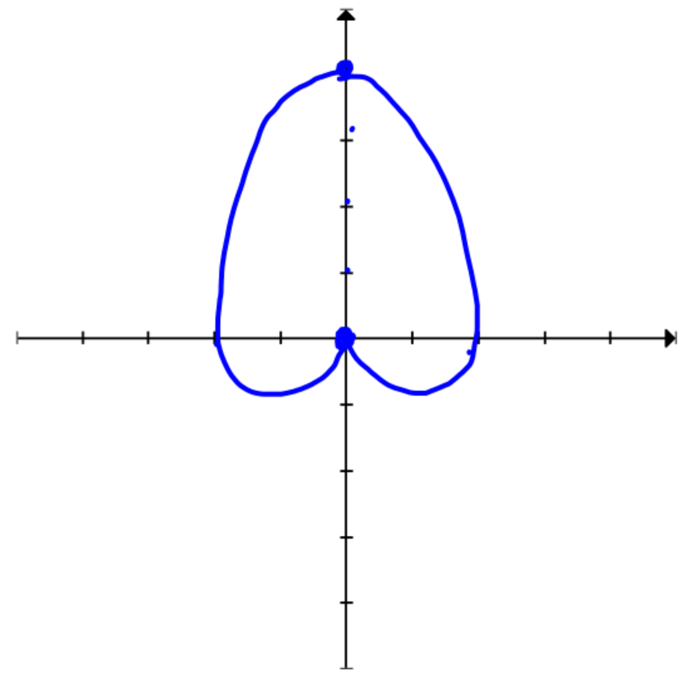
at origin when $r = 0$

$0 = 2 + 2\sin\theta$

$-2 = 2\sin\theta$

$-1 = \sin\theta$

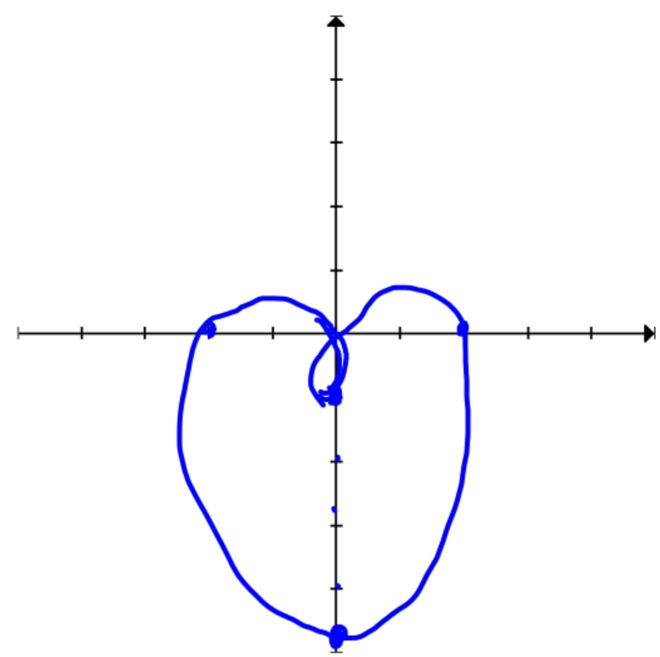
$\theta = \frac{3\pi}{2}$



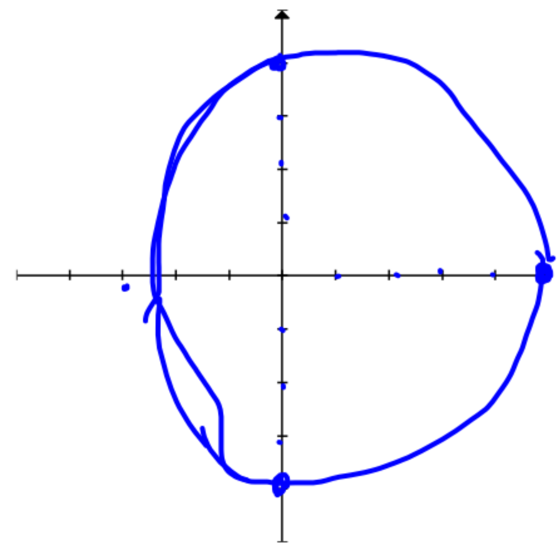
Graph: $r = 2 - 3\sin\theta$

$0 = 2 - 3\sin\theta$

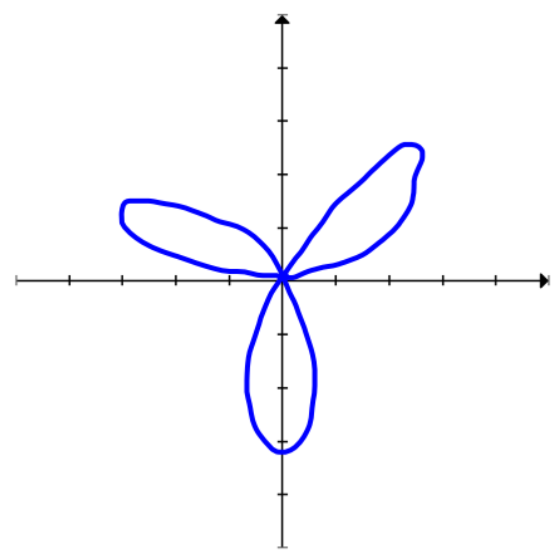
$\frac{2}{3} = \sin\theta$



Graph: $r = 4 + 1\cos\theta$



Graph: $r = 3\sin 3\theta$

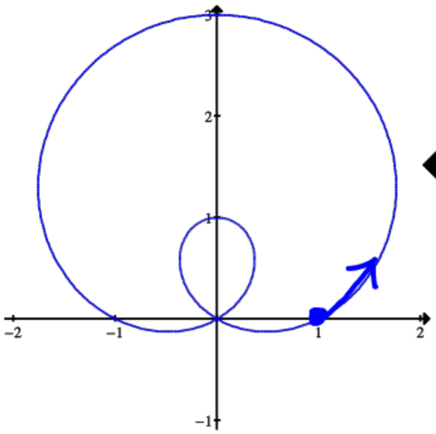


More about polar graphs:

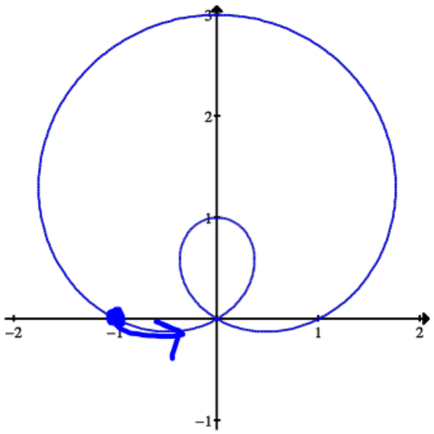
Some Limaçons with an inner loop: Sine plots align along the y axis

$r = 1 + 2sin(\theta)$

$r = -1 + 2sin(\theta)$

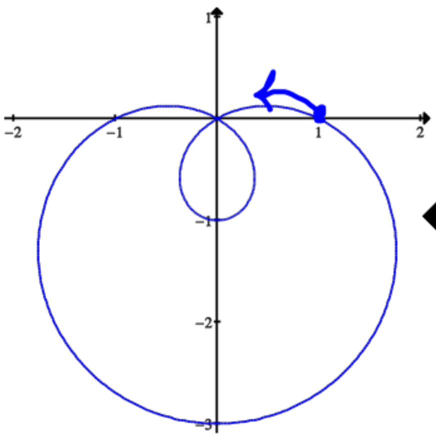


← Identical plots, but different starting points with $\theta = 0$. →



$r = 1 - 2sin(\theta)$

$r = -1 - 2sin(\theta)$



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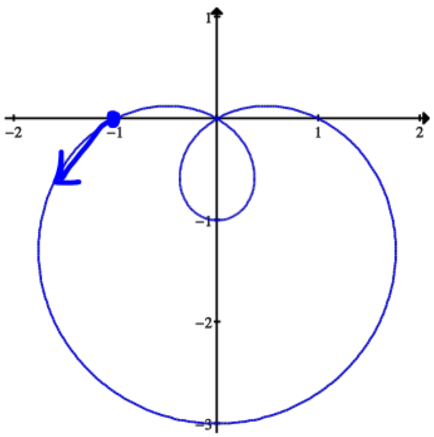
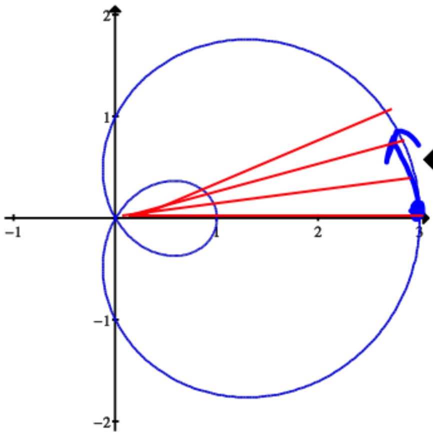


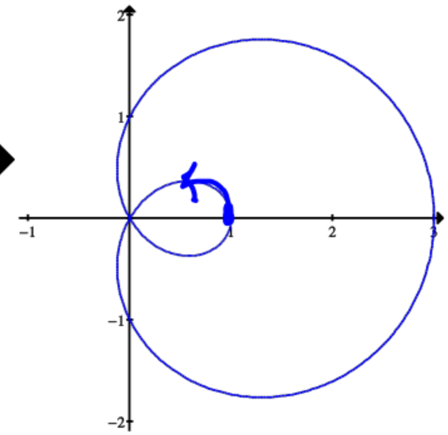
More Limaçons with an inner loop: Cosine plots align along the x axis

$r = 1 + 2cos(\theta)$

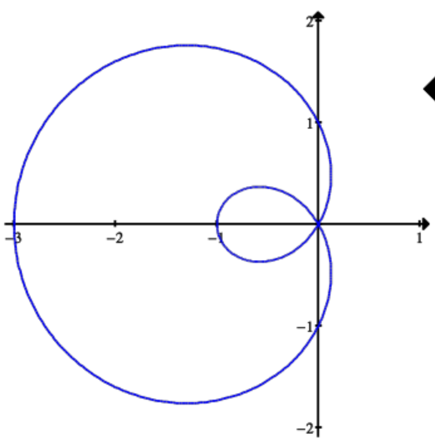


← Identical plots, but different starting points with $\theta = 0$. →

$r = -1 + 2cos(\theta)$

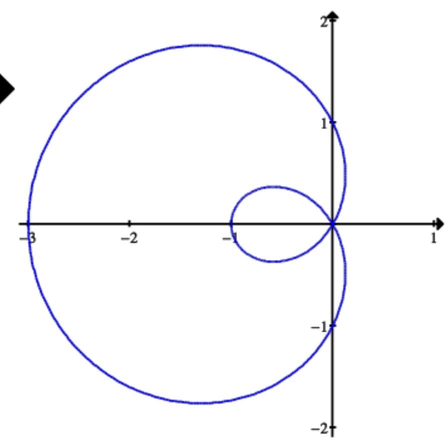


$r = 1 - 2cos(\theta)$



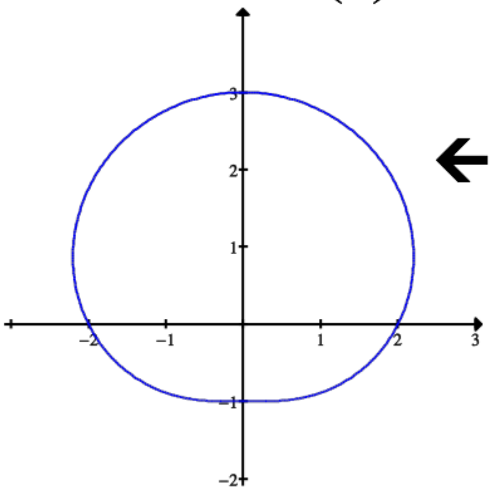
← Identical plots, but different starting points with $\theta = 0$. →

$r = -1 - 2cos(\theta)$

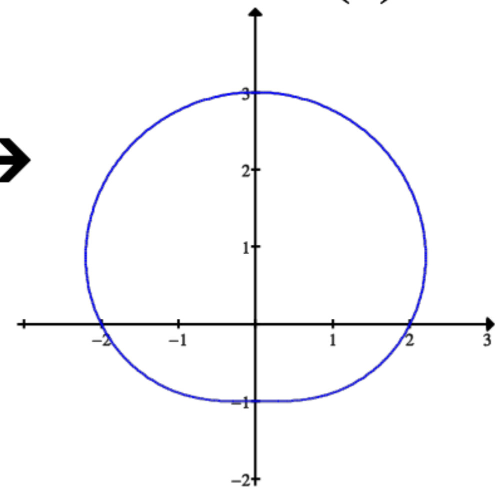


Some Limaçons with a dent (dimple): Sine plots align along the y axis

$r = 2 + 1\,sin(\theta)$

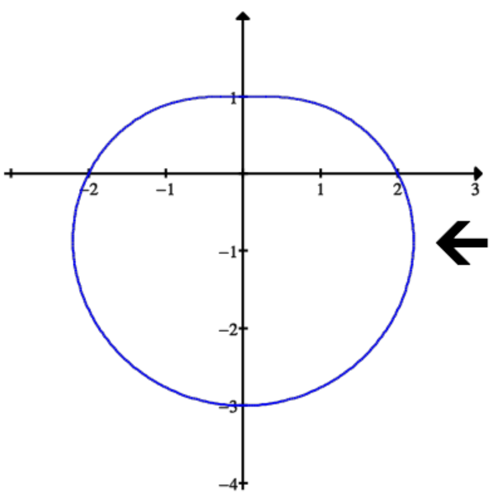


$r = -2 + 1\,sin(\theta)$

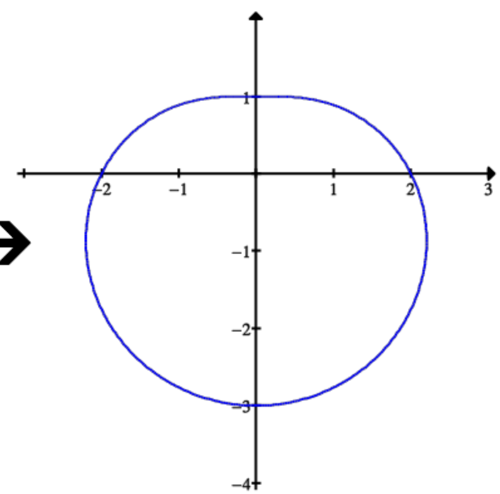


Identical plots, but different starting points with $\theta = 0$.

$r = 2 - 1\,sin(\theta)$



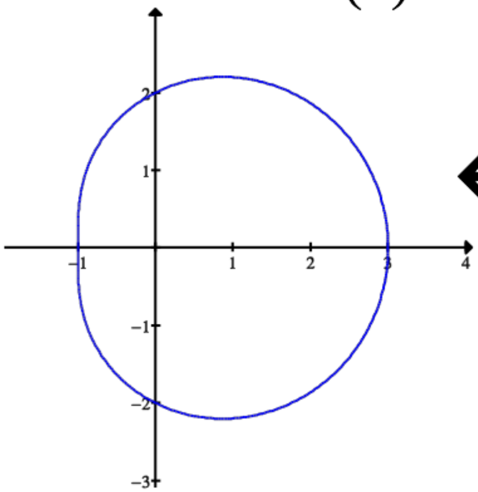
$r = -2 - 1\,sin(\theta)$



Identical plots, but different starting points with $\theta = 0$.

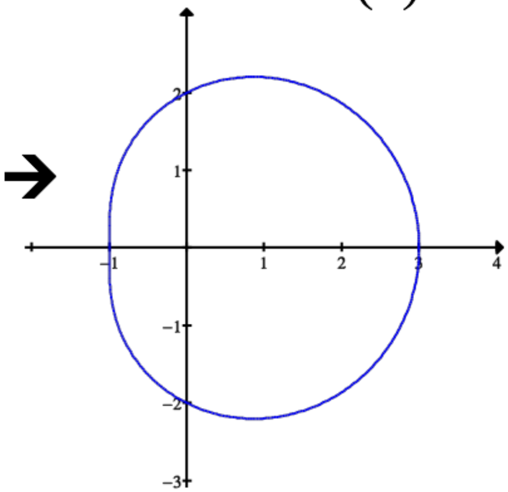
More Limaçons with a dent (dimple): Cosine plots align along the x axis

$r = 2 + 1\cos(\theta)$

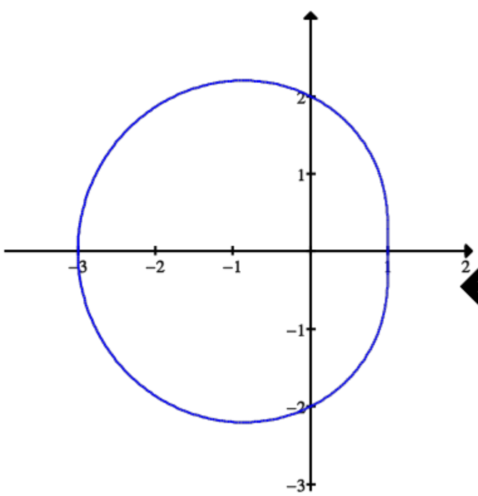


Identical plots, but different starting points with $\theta = 0$.

$r = -2 + 1\cos(\theta)$

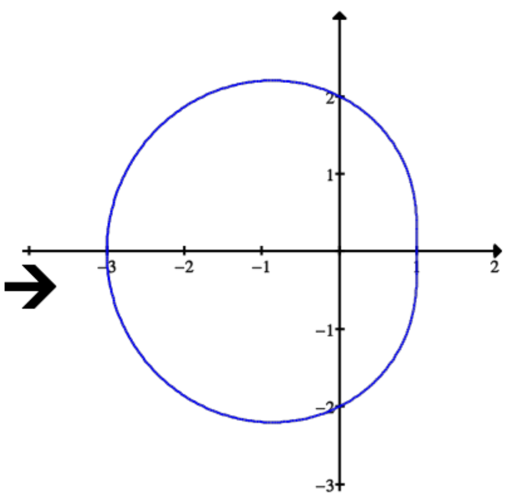


$r = 2 - 1\cos(\theta)$



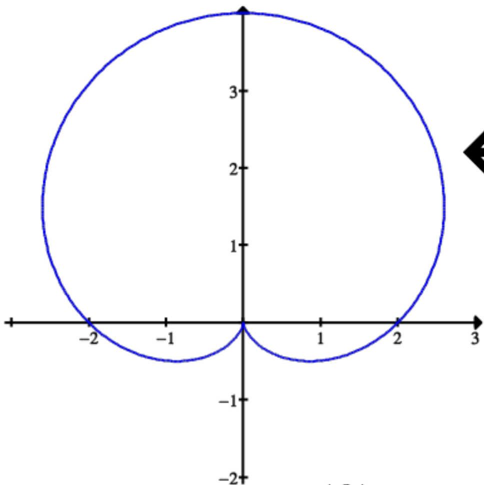
Identical plots, but different starting points with $\theta = 0$.

$r = -2 - 1\cos(\theta)$

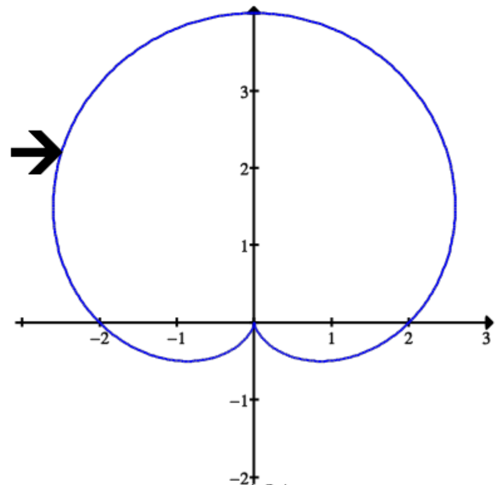


Some cardioids: Sine plots align along the y axis

$r = 2 + 2\sin(\theta)$

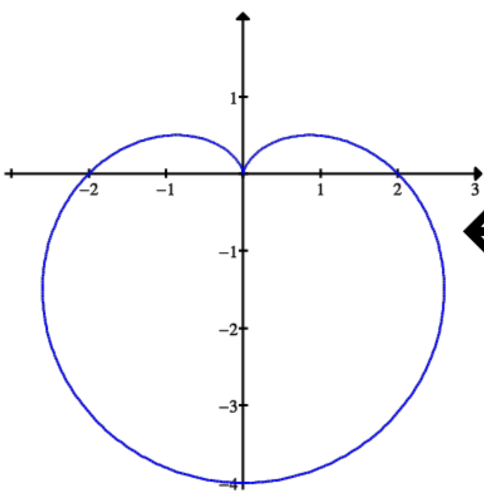


$r = -2 + 2\sin(\theta)$

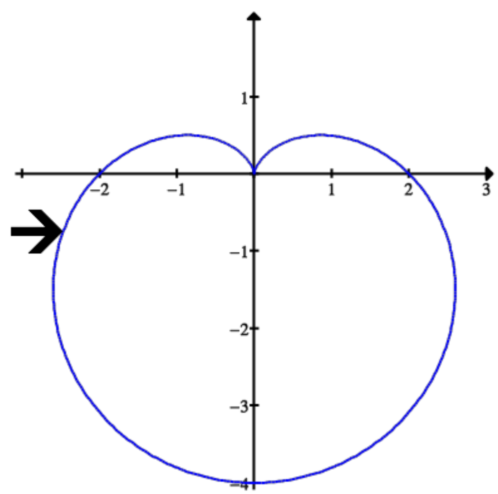


Identical plots, but different starting points with $\theta = 0$.

$r = 2 - 2\sin(\theta)$



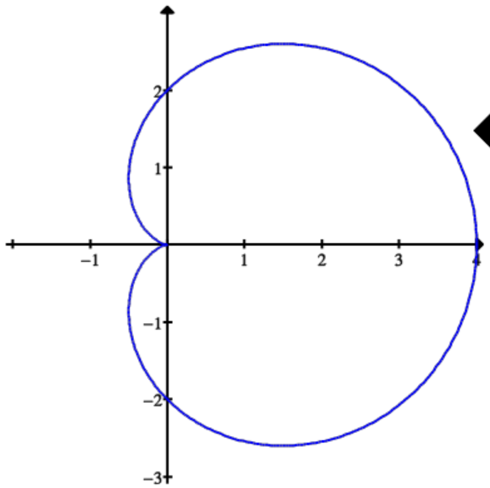
$r = -2 - 2\sin(\theta)$



Identical plots, but different starting points with $\theta = 0$.

More cardioids: Cosine plots align along the x axis

$r = 2 + 2cos(\theta)$



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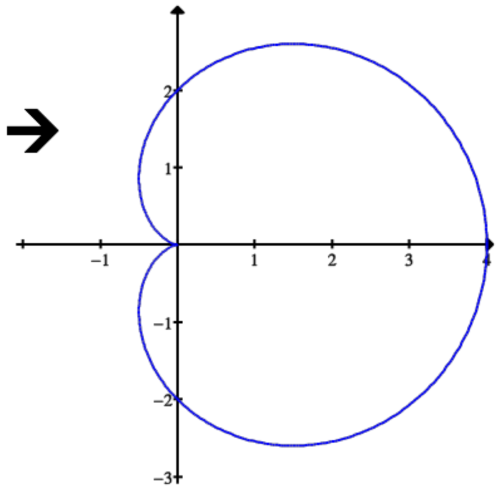
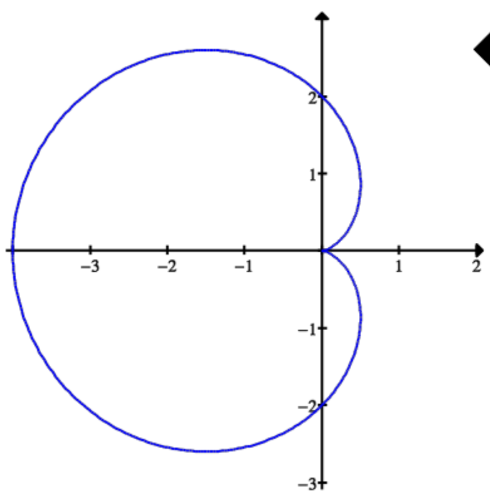
$r = -2 + 2cos(\theta)$



$r = 2 - 2cos(\theta)$



← Identical plots, but different starting points with $\theta = 0$. →

$r = -2 - 2cos(\theta)$