Math 1432

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Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Popper 32

1. Which of the following is the cardioid?

a. $r = 3 - 3\ \mathit{cos}\ \theta$

b. $r = 4 + 5\ \mathit{sin}\ \theta$

c. $r = 4 + 3\ \mathit{cos}\ \theta$

d. $r = 2\ \mathit{sin}\ \theta$

e. $r = 4\ \mathit{cos}\ (4\theta)$

2. Which of the following is the flower?

a. $r = 3 - 3\,cos\,\theta$

b. $r = 4 + 5\,sin\,\theta$

c. $r = 4 + 3\,cos\,\theta$

d. $r = 2\,sin\,\theta$

e. $r = 4\,cos\,(4\theta)$

3. Which of the following is the limaçon with a dent (dimple)?

a. $r = 3 - 3\,cos\,\theta$

b. $r = 4 + 5\,sin\,\theta$

c. $r = 4 + 3\,cos\,\theta$

d. $r = 2\,sin\,\theta$

e. $r = 4\,cos\,(4\theta)$

4. Which of the following is the limaçon with an inner loop?

a. $r = 3 - 3\,cos\,\theta$

b. $r = 4 + 5\,sin\,\theta$

c. $r = 4 + 3\,cos\,\theta$

d. $r = 2\,sin\,\theta$

e. $r = 4\,cos\,(4\theta)$

5. Which of the following is the circle?

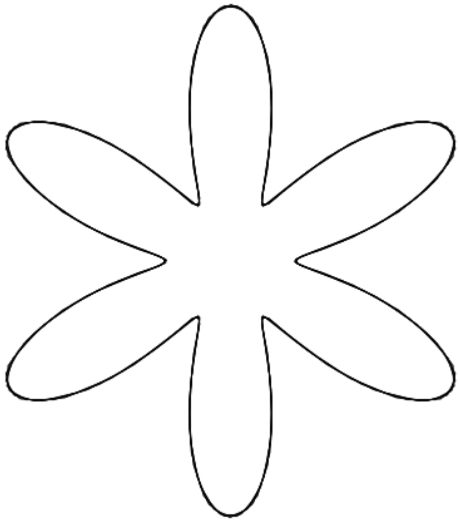
a. $r = 3 - 3 \cos \theta$

b. $r = 4 + 5 \sin \theta$

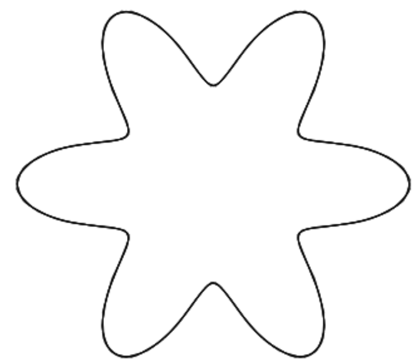
c. $r = 4 + 3 \cos \theta$

d. $r = 2 \sin \theta$

e. $r = 4 \cos (4\theta)$



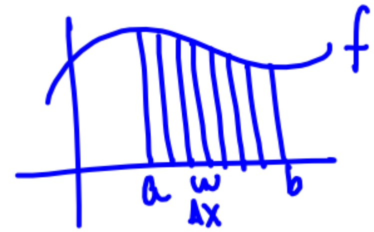
Area in Polar Coordinates



The area of a polar region is based on the area of a sector of a circle.

Area of a circle = πr^2

$$\int_a^b f(x)\,dx$$

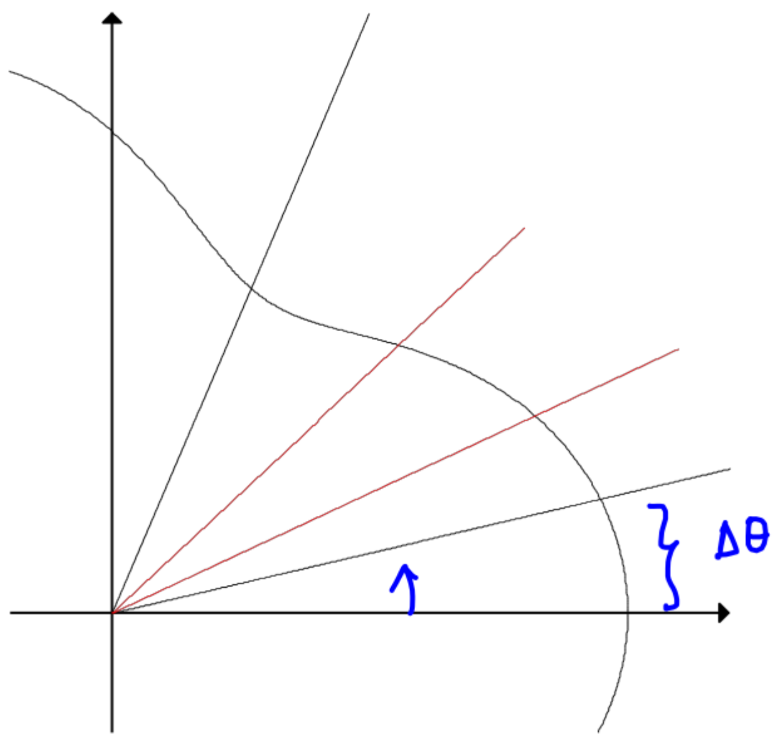


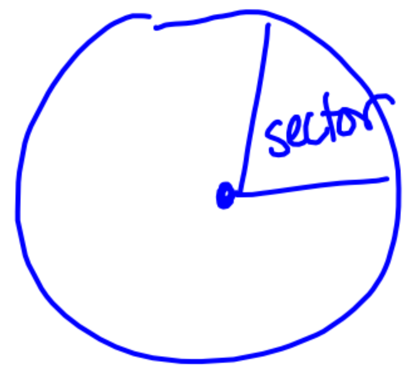
Therefore the area of a sector of a circle is the part of the circle you want times the area of the whole circle:

$$\text{Area sector} = \frac{\theta}{2\pi} \bullet \pi r^2 = \frac{1}{2} r^2 \theta$$

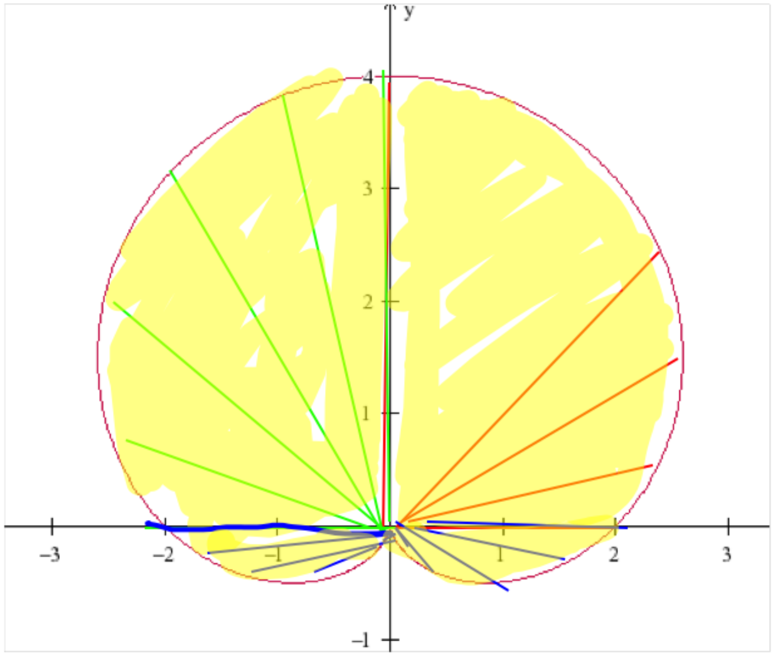
$$\int_\alpha^\beta \frac{1}{2} r^2 \, d\theta$$

Find the area of the region between the origin and the polar graph of $r = \rho(\theta)$ for θ between a and b.





1. Find the area bounded by the graph of $r = 2 + 2\,sin\,\theta$.



$$\int_{\boxed{0}}^{\boxed{2\pi}} \frac{1}{2}(2+2\sin\theta)^2\,d\theta$$

$$\int_0^{2\pi} \frac{1}{2}(4 + 8\sin\theta + 4\sin^2\theta)\,d\theta$$

$$\int_0^{2\pi} (2 + 4\sin\theta + 2\sin^2\theta)\,d\theta$$

$$= 2\theta - 4\cos\theta + 2\left(\frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta\right)\Big|_0^{2\pi}$$

$$= 2(2\pi) - 4 + 2\left(\frac{1}{2}(2\pi) - \frac{1}{2}(0)(1)\right) - [0 - 4 + 0]$$

$$4\pi - 4 + 2\pi + 4 = 6\pi$$

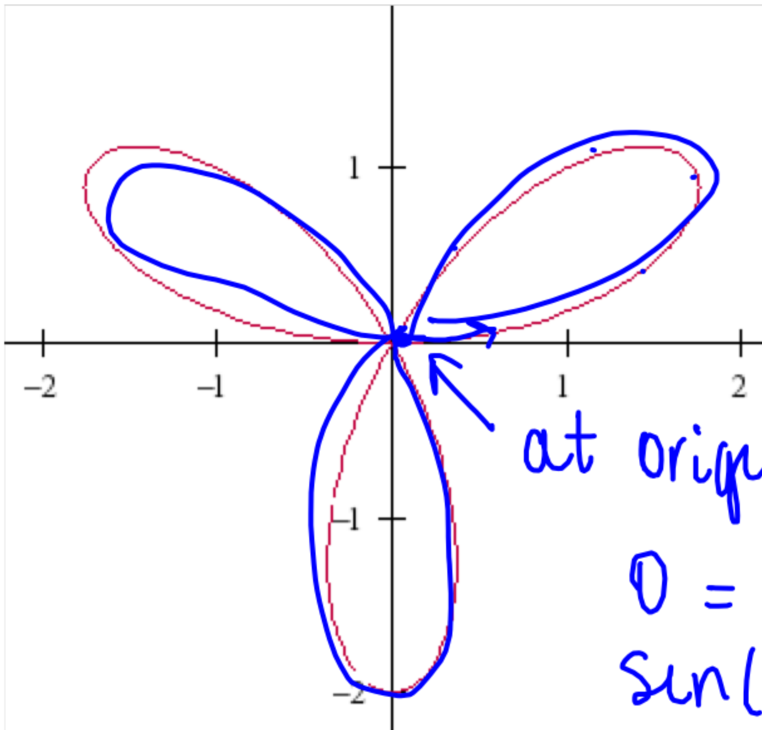
$$\int \sin^2\theta \, d\theta = \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + C$$

$$\int \cos^2\theta \, d\theta = \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$

2. Find the area inside one petal of the flower given by $r = 2\,sin\,(3\theta)$.

$$\boxed{\int_0^{\pi/3} \frac{1}{2}\left(2\sin(3\theta)\right)^2 d\theta}$$

θ	r
0	0



at origin when $r=0$

$0 = 2\sin 3\theta$

$\sin(3\theta) = 0$

$3\theta = 0, \pi, 2\pi, 3\pi, \ldots$

$\theta = \underbrace{0, \pi/3}, 2\pi/3, \pi, \ldots$

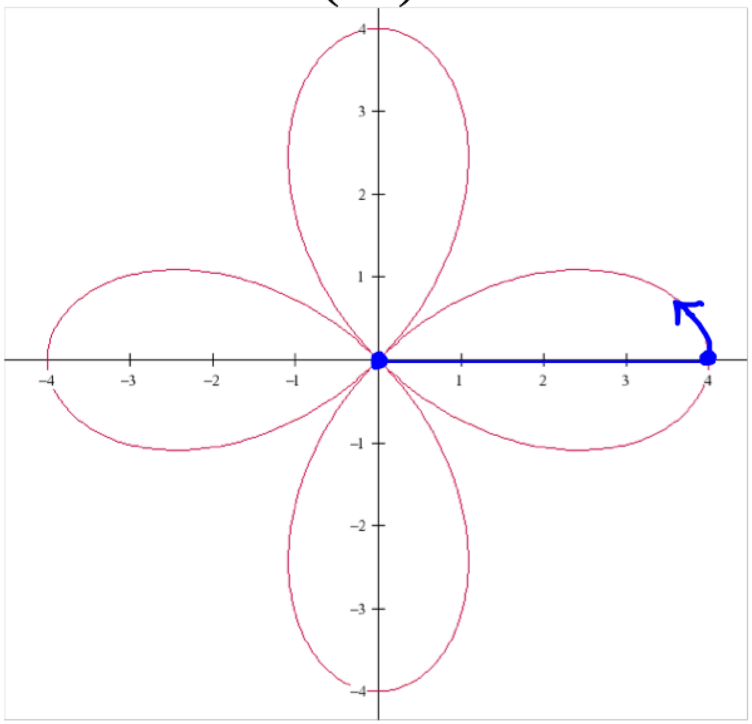
6. SHHHHH

Are you paying attention? Don’t share the answer to this one.

a. yes ← ← ← this is the correct answer

b. no

3. Find the area inside one petal of the flower given by
$r = 4\ cos\ (2\theta)$.



$0 = 4\cos(2\theta)$

$\cos(2\theta) = 0$

$2\theta = \pi/2,\ 3\pi/2,\ 5\pi/2, \ldots$

$\theta = \underbrace{\pi/4,\ 3\pi/4}, 5\pi/4, \ldots$

$$2\int_0^{\pi/4} \frac{1}{2}(4\cos(2\theta))^2\, d\theta$$

or

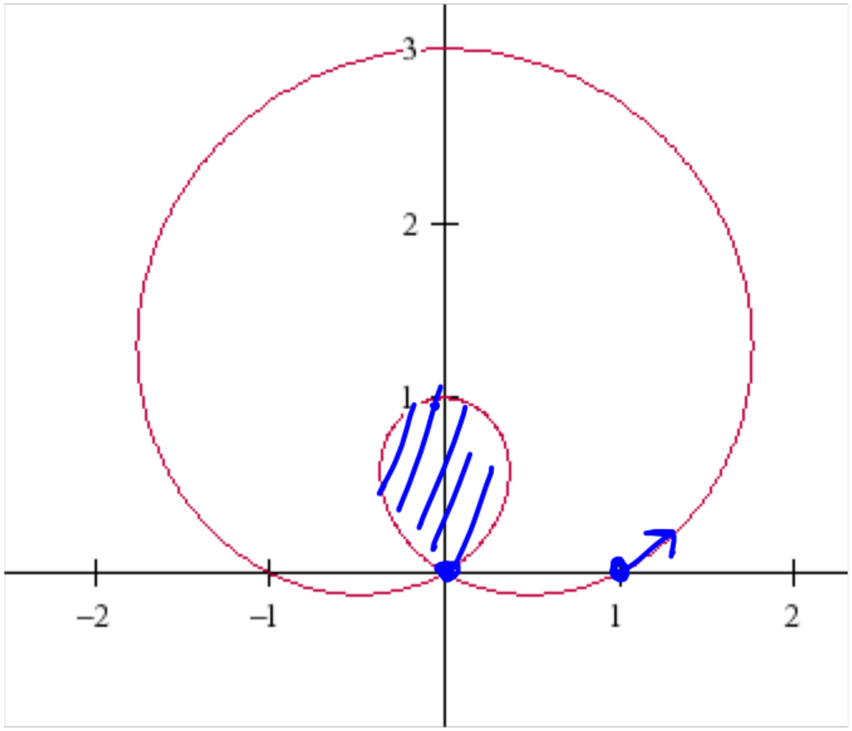
$$\int_{\pi/4}^{3\pi/4} \frac{1}{2}(4\cos(2\theta))^2\, d\theta$$

each petal has period of $\pi/2$

$4 \cdot \pi/2 = 2\pi$

entire flower $\int_0^{2\pi} \frac{1}{2}(4\cos(2\theta))^2\, d\theta$

4. Find the area inside THE INNER LOOP of $r = 1+2\sin\theta$



$r = 0 \Rightarrow 0 = 1 + 2\sin\theta$

$-\frac{1}{2} = \sin\theta$

$\theta = 7\pi/6,\ 11\pi/6$

$$\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}(1+2\sin\theta)^2\, d\theta$$

$\theta = 0 \rightarrow 1 + 0 = 1$

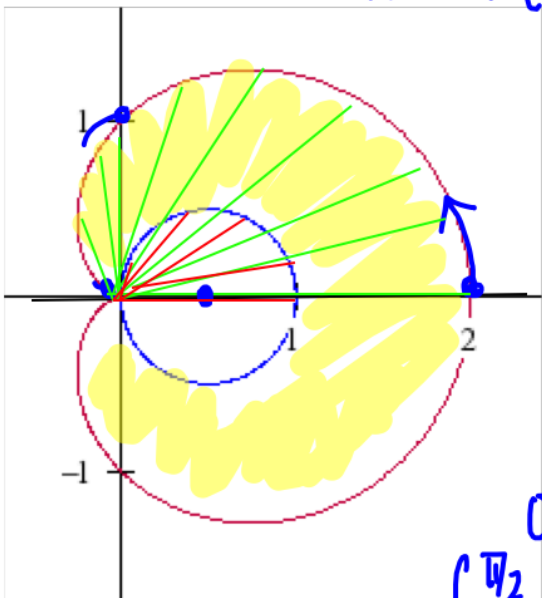
5. Write the integral to find the area between r = 1 + *cos* θ,

$\boxed{r = \cos\theta,}$ ~~for θ = 0 to θ = π/2~~

$A = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$

$\theta = 0 \rightarrow r = 2$

$\theta = \pi \rightarrow r = 0$



$$2\int_0^{\pi} \frac{1}{2}(1+\cos\theta)^2\, d\theta - \frac{\pi}{4}$$

or

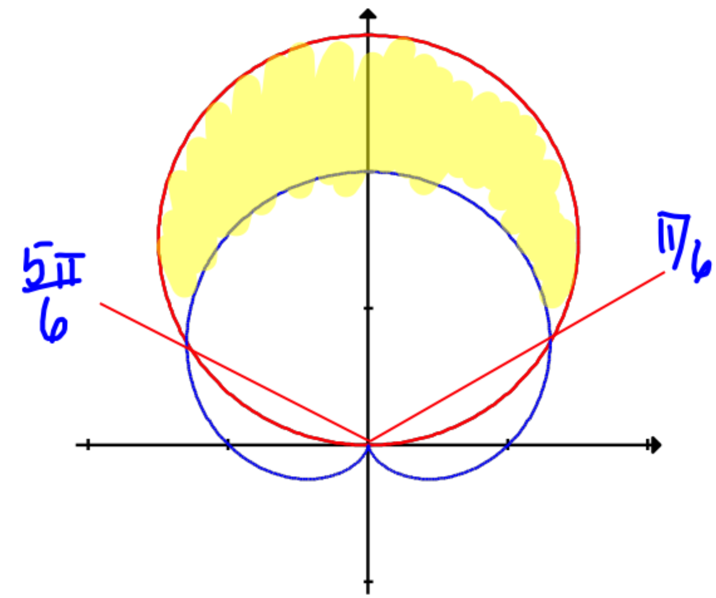
$$2\int_0^{\pi/2} \frac{1}{2}(\cos\theta)^2\, d\theta$$

6. Find the area inside r = 3 *sin* θ and outside r = 1 + *sin* θ.

$d = 3$

$r = 3/2$

$A = \pi(3/2)^2$



$3\sin\theta = 1 + \sin\theta$

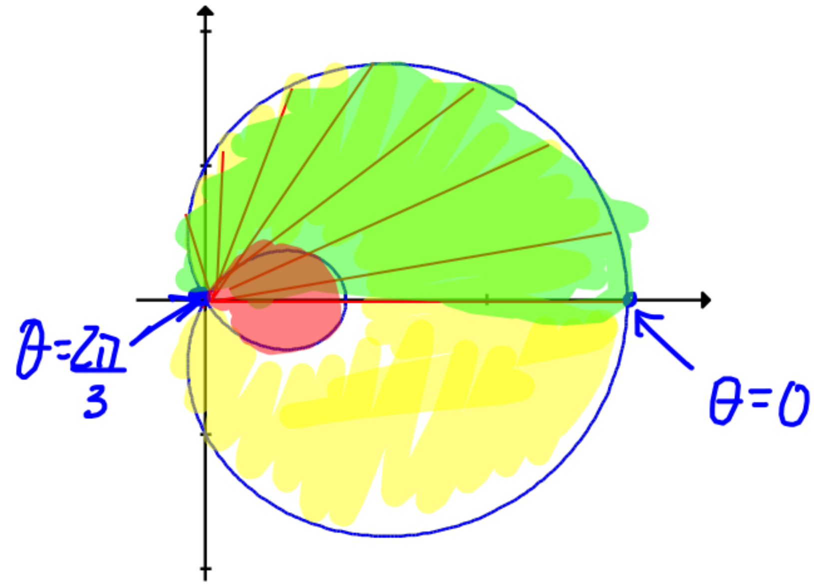
$2\sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \pi/6, 5\pi/6$

$$\int_{\pi/6}^{5\pi/6} \underbrace{\tfrac{1}{2}(3\sin\theta)^2}_{\text{circle}} - \underbrace{\tfrac{1}{2}(1+\sin\theta)^2}_{\text{cardiod}}\, d\theta$$

7. Find the area between the loops of $r = 1 + 2\cos\theta$.



Inner loop

$0 = 1 + 2\cos\theta$

$-\frac{1}{2} = \cos\theta$

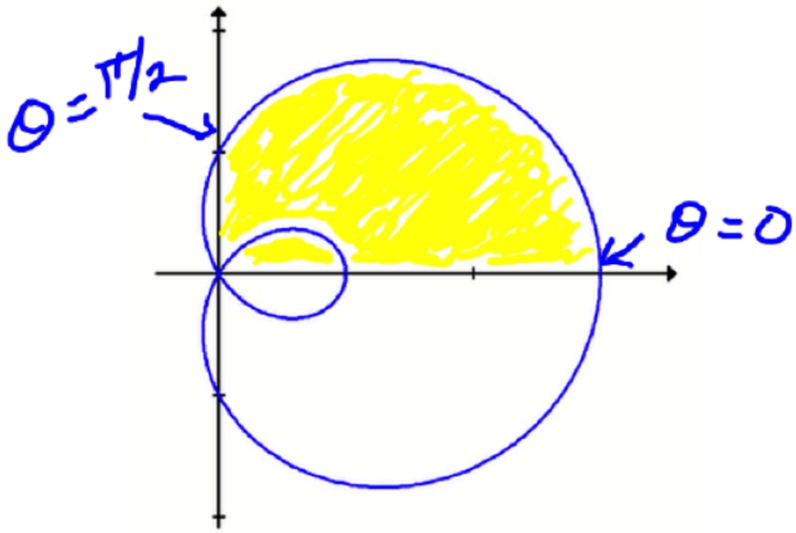
$\theta = 2\pi/3,\ 4\pi/3$

Entire region: $2\int_0^{2\pi/3} \frac{1}{2}(1+2\cos\theta)^2\,d\theta$

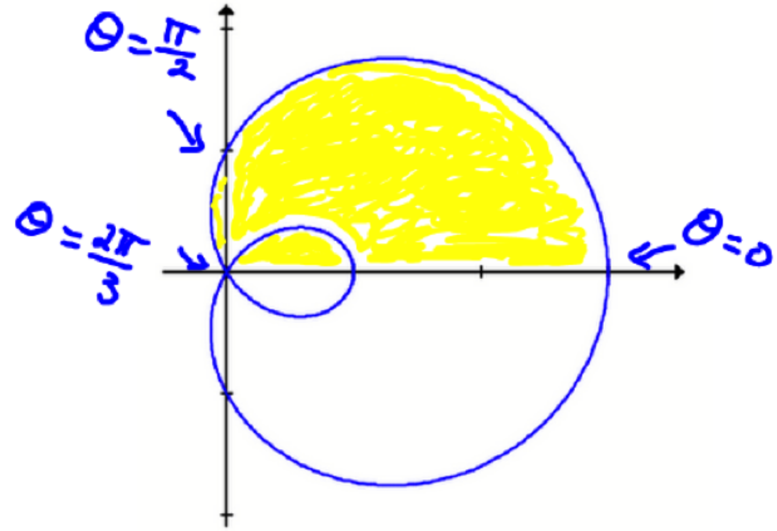
Inner Loop: $\int_{2\pi/3}^{4\pi/3} \frac{1}{2}(1+2\cos\theta)^2\,d\theta$

ans: $2\int_0^{2\pi/3} \frac{1}{2}(1+2\cos\theta)^2\,d\theta - \int_{2\pi/3}^{4\pi/3} \frac{1}{2}(1+2\cos\theta)^2\,d\theta$

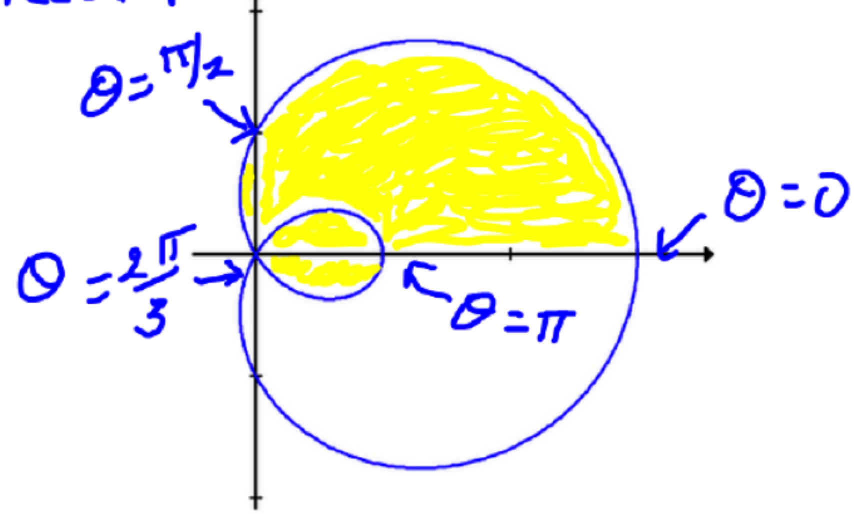
area from 0 to $\pi/2$



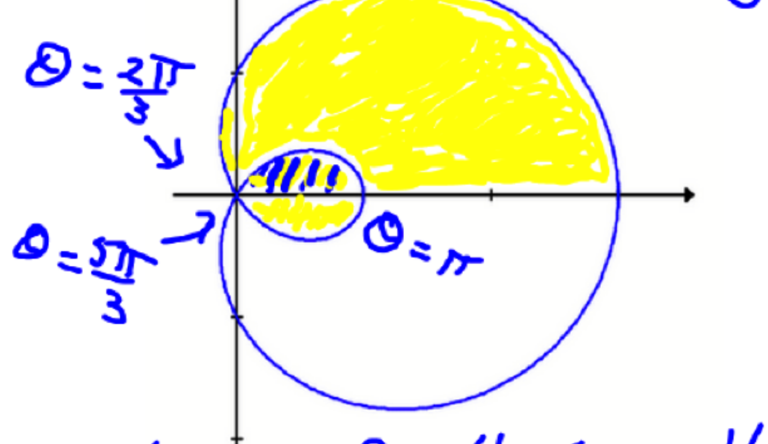
area from 0 to $\frac{2\pi}{3}$



area from 0 to π

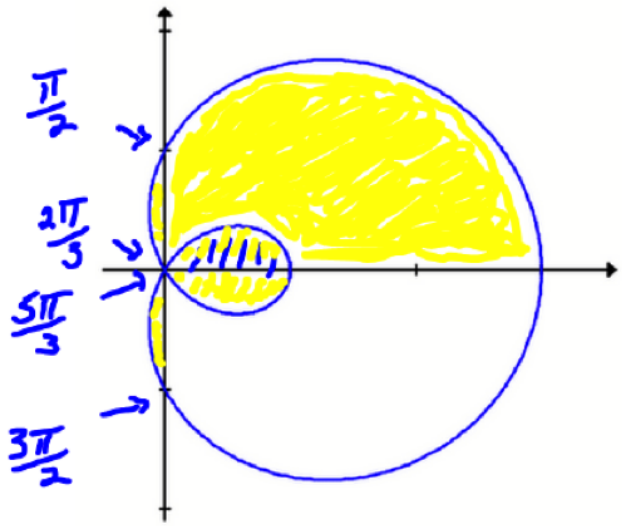


area traced from 0 to $\frac{5\pi}{3}$



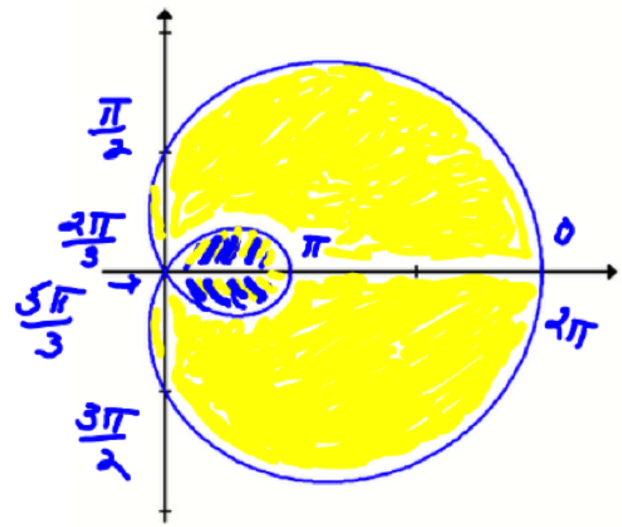
//// traced for the second time

area traced from 0 to $3\pi/2$



|||| traced twice

area traced from 0 to 2π

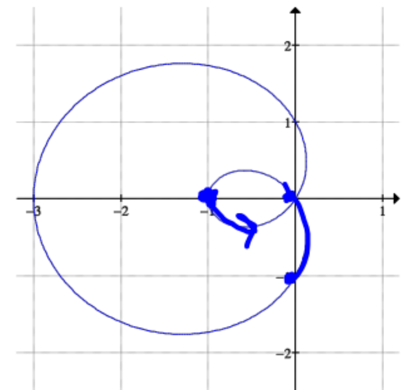


//// + \\\\ traced twice

8.

Give the area of the region that is in quadrant 4 and inside the outer loop of the polar graph

$r = 1 - 2\,cos\,(\theta)$

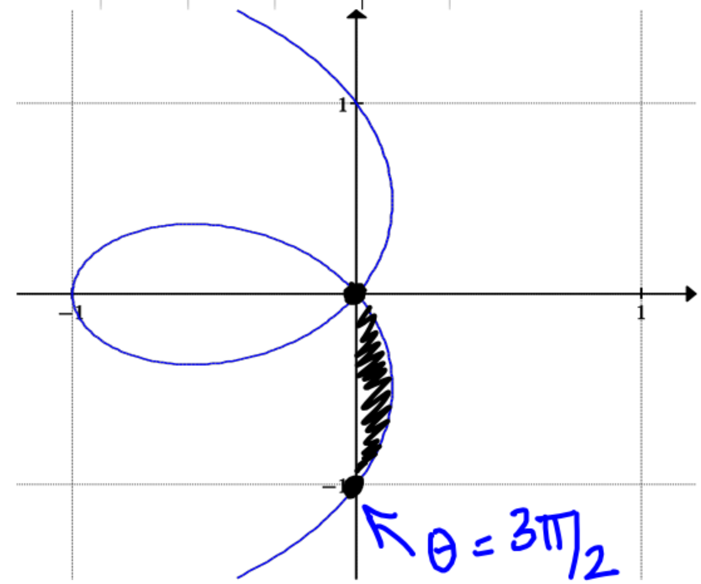


$0 = 1 - 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \pi/3,\ 5\pi/3$

$$\int_{\frac{3\pi}{2}}^{\frac{5\pi}{3}} \frac{1}{2}\left(1-2\cos\theta\right)^2 d\theta$$



7. Give the integral that will determine the area inside one petal of the flower given by r = *sin* (3θ).

$0 = \sin(3\theta)$

$3\theta = 0, \pi, 2\pi, \ldots$

$\theta = \underbrace{0, \pi/3}, 2\pi/3, \ldots$

a. $\frac{1}{2}\int_0^{\frac{\pi}{3}} (\sin 3\theta)\,d\theta$

b. $\int_0^{\frac{\pi}{3}} (\sin 3\theta)^2\,d\theta$

c. $\frac{1}{2}\int_0^{\frac{\pi}{3}} (\sin 3\theta)^2\,d\theta$

~~d.~~ $2\int_0^{\frac{\pi}{6}} (\sin 3\theta)^2\,d\theta$

~~e.~~ $\frac{1}{2}\int_0^{\frac{\pi}{6}} (\sin 3\theta)^2\,d\theta$

How can we find the length of a polar curve?

$$L(c) = \int_{\alpha}^{\beta} \sqrt{[\rho(\theta)]^2 + [\rho'(\theta)]^2}\, d\theta$$

$r \qquad r'$

Verify the formula for the circumference of a circle with radius a using the formula above.

$2\pi a$

$r = a$

$r' = 0$

$$\int_0^{2\pi} \sqrt{a^2 + 0^2}\, d\theta = \int_0^{2\pi} a\, d\theta$$

$$= 2\pi a$$

Set up the integral to find the length of one petal of the curve $r = \cos 3\theta$

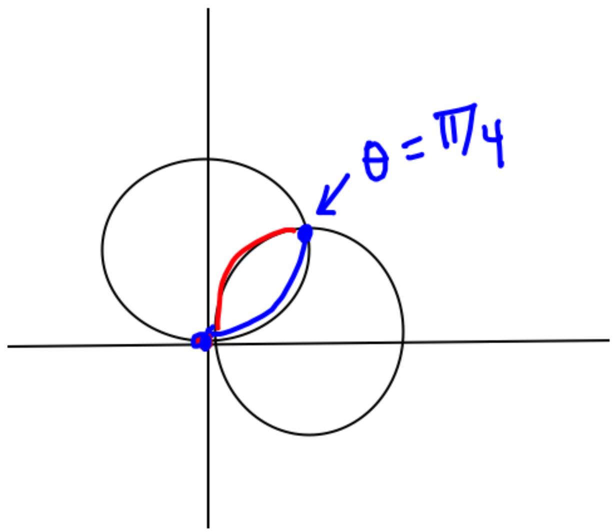
$$0 = \cos 3\theta$$

$$3\theta = \pi/2,\ 3\pi/2,\ \ldots$$

$$\theta = \pi/6,\ \pi/2$$

$$\int_{\pi/6}^{\pi/2} \sqrt{\underbrace{(\cos 3\theta)^2}_{r} + \underbrace{(-3\sin 3\theta)^2}_{r'}}\ d\theta$$

Determine the length of the perimeter of the region in Quadrant I bounded by the circles $r=2\sin\theta$ and $r=2\cos\theta$



$$\int_0^{\pi/4} \sqrt{(2\sin\theta)^2 + (2\cos\theta)^2}\, d\theta$$

$$+$$

$$\int_{\pi/4}^{\pi/2} \sqrt{(2\cos\theta)^2 + (-2\sin\theta)^2}\, d\theta$$

$$= \int_0^{\pi/4} \sqrt{4\underbrace{(\sin^2\theta + \cos^2\theta)}_{=1}}\, d\theta \; +$$

$$= \int_0^{\pi/4} 2\, d\theta + \int_{\pi/4}^{\pi/2} 2\, d\theta$$

$$2(\pi/4) + 2(\pi/2) - 2(\pi/4) = \boxed{\pi}$$