Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

More IBP examples:

$$\int u\,dv = uv - \int v\,du$$

1. $\int_0^{\pi/2} x^2 \sin x\,dx$

2. $\int \left(e^{x} + 2x \right)^{2} dx$

3. $\int x^2 \arctan x \, dx$

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1. $\int xe^{x}\,dx =$

2. Rewrite $\cos^{2} x$ in terms of sine.

8.2 Powers and Products of Trigonometric Functions

Recall the following identities:

$$\cos^2(x) + \sin^2(x) = 1$$
$$1 + \tan^2(x) = \sec^2(x)$$
$$1 + \cot^2(x) = \csc^2(x)$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2} \qquad \sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

In this section, we will study techniques for evaluating integrals of the form

$$\int \sin^m x \cos^n x \, dx \qquad \text{and} \qquad \int \sec^m x \tan^n dx$$

where either m or n is a positive integer.

To find antiderivatives for these forms, try to break them into combinations of trigonometric integrals to which you can apply the Power Rule, which is

$$\int u^n \, du = \begin{cases} \dfrac{u^{n+1}}{n+1} + C & \text{if } \; n \neq 1 \\ \ln|u| + C & \text{if } \; n = -1 \end{cases}.$$

Integrals Involving Powers of Sine and Cosine

1. **If *m* or *n* odd:**
   a. *m* odd: rewrite $\sin^m x$ as $\sin^{m-1} x \sin x$ (*m*-1 is even so can use identity $\sin^2 x = 1 - \cos^2 x$)
   b. *n* odd: rewrite $\cos^m x$ as $\cos^{m-1} x \cos x$ (*n*-1 is even so can use identity $\cos^2 x = 1 - \sin^2 x$)

Examples:

$$\int \sin^3 x \, dx$$

$$\int \sin^3 x \cos^2 x \, dx$$

$$\int \cos^5 x \, dx$$

2. **If *m* and *n* even use these identities:**

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1+\cos 2x}{2}.$$

$$\int \cos^2 x \, dx$$

Note:

$$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{2}\sin x\cos x + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{2}\sin x\cos x + C$$

Integrals involving Secants and Tangents

$$\tan^2 x + 1 = \sec^2 x$$

For $\int \tan^m x \sec^n x dx$

a. n even: rewrite $\tan^m x \sec^n x$ as $\tan^m x \sec^{n-2} x \sec^2 x$ (then you can use identity $\sec^2 x = \tan^2 x + 1$)

b. m odd: rewrite as $\tan^{m-1} x \sec^{n-1} x \cdot \sec x \tan x$ (m-1 is even so can use identity $\tan^2 x = \sec^2 x - 1$)

c. m even and n odd: rewrite $\tan^m x$ using $\tan^2 x = \sec^2 x - 1$

Examples:

$$\int \tan^3(x)\,dx$$

$$\int \sec^4 x \, dx$$

$$\int \sec^4 x \tan^2 x \, dx$$

Note:

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx \quad n \geq 2$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx \quad n \geq 2$$

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3. $\int \cos x \sin^3 x \, dx$

4. Compute $\int \sec(2x)\tan^3(2x)\,dx$