Math 1432

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Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Compute: $\int \frac{3x^3 - 2}{x^2 + 4} dx$

Partial Fractions:

Example:
$$\frac{3}{x} + \frac{4}{x+1} = \frac{3(x+1)+4x}{x(x+1)} = \frac{7x+3}{x(x+1)}$$

What if we have $\frac{7x+3}{x(x+1)}$ and want the original two fractions?

$$\frac{7x+3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{3}{x} + \frac{4}{x+1}$$

How do we find A and B?

In general, each linear factor of the form $(x - \alpha)^k$ in the denominator gives rise to an expression of the form

$$\frac{A_1}{\left(x-a\right)} + \frac{A_2}{\left(x-a\right)^2} + \ldots + \frac{A_k}{\left(x-a\right)^k}$$

Give the form of the partial fraction decomposition for:

1a.
$$\frac{5x^2 - 6x + 1}{(x+1)(x+2)(x+3)}$$

1b.
$$\frac{2x^2 - 3x + 1}{(x+1)(x+2)^2(x+3)}$$

Rewrite using partial fractions:

$$\frac{5x-10}{(x-4)(x+1)}$$

$$\frac{3x^2 + 20x + 25}{(x-1)(x+2)(x+3)}$$

Popper 13

1. Give the form of the partial fraction decomposition for:

$$\frac{3x^2 + 20x + 25}{(x-1)(x+2)(x+3)^2}$$

Compute:

$$\int \frac{4x + 41}{x^2 + 3x - 10} \, dx$$

$$\int \frac{x^2 + 1}{x(x^2 - 1)} dx$$

$$\int \frac{x^2 + 3}{x^2 - 3x + 2} dx$$

$$\int \frac{dx}{x^2 + x - 2}$$

$$\int \frac{x+1}{x^3 - x^2} dx$$

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2. Give the form of the PFD for: $\frac{1}{x^2 - x - 6}$

3. Find the PFD for:
$$\frac{1}{x^2 - x - 6}$$

$$4. \text{Compute: } \int \frac{1}{x^2 - x - 6} dx$$

Now, if the denominator has an irreducible quadratic factor of the form

$$x^{2} + bx + c$$
 then we have a term in the form $\frac{Ax + B}{x^{2} + bx + c}$

Give the form of the PFD for the following:

$$\frac{1}{x(x^2+1)}$$

$$\frac{1}{(x-2)^2(x^2+2)}$$

$$\frac{1}{(x^2-1)(x^2+4)}$$

5. Give the form of the PFD for: $\frac{1}{(x-2)^2(x^2+2)}$

$$\frac{1}{(x-2)^2(x^2+2)}$$

More examples. Give the PFD and then integrate:

$$\int \frac{1}{x(x^2+1)} dx$$

$$\int \frac{x^2 + 2x + 7}{(x-1)^2(x^2 + 4)} dx$$

$$\int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

6.If the degree in the numerator of a rational expression is equal or higher than the degree in the denominator, you should do this first: