# Math 1432

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Office Hours:

Mondays 1-2pm,
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## Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

# Simpson's Method

Fit a parabola to every section.

$$S_n = \frac{b-a}{3n}\left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

**n must be even**

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Use Simpson's method with $n = 6$ to approximate $\int_0^1 \frac{1}{x+1}\,dx$.

**Theoretical error - Simpson's Rule:** The theoretical error of Simpson's Rule

$$S_n^T = \int_a^b f(x)\,dx - S_n$$

is given by

$$S_n^T = \frac{(b-a)^5}{180n^4}\, f^{(4)}(c)$$

for some $c \in (a,b)$. As above, we usually do not know $c$, but, if $f^{(4)}$ is bounded on $[a,b]$, say $\left|f^{(4)}(x)\right| \le M$ for all $x \in [a,b]$, then

$$\left|S_n^T\right| \le \frac{(b-a)^5}{180n^4}\, M.$$

Give a value of n that will guarantee Simpson's method approximates $\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$ within $10^{-4}$. $\left|E_n^S\right| \le \frac{(b-a)^5}{180n^4} M$ where $\left|f^{(4)}(x)\right| \le M$ for $a \le x \le b$.

$$f(x) = \sin 2x$$
$$f'(x) = 2\cos 2x$$
$$f''(x) = -4\sin 2x$$
$$f'''(x) = -8\cos 2x$$
$$f^{(4)}(x) = 16\sin 2x$$

# General Formulas to approximate

$$\int_a^b f(x)\,dx$$

## Left Hand Endpoint Method:

$$L_n = \frac{b-a}{n}\left[ f(x_0) + f(x_1) + \cdots + f(x_{n-1}) \right]$$

## Right Hand Endpoint Method:

$$R_n = \frac{b-a}{n}\left[ f(x_1) + f(x_2) + \cdots + f(x_n) \right]$$

## Midpoint Method:

$$M_n = \frac{b-a}{n}\left[ f\left(\frac{x_0 + x_1}{2}\right) + \cdots + f\left(\frac{x_{n-1} + x_n}{2}\right) \right]$$

Trapezoid Method:

$$T_n = \frac{b-a}{2n}\left[ f\left(x_0\right) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f\left(x_n\right)\right]$$

Simpson's Rule:

$$S_n = \frac{b-a}{3n}\left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

Error Estimate

Trapezoid: $E_n^T = \frac{(b-a)^3}{12n^2} f''(c), \quad \left|E_n^T\right| \le \frac{(b-a)^3}{12n^2} M$

Simpson's: $E_n^S = \frac{(b-a)^5}{180n^4} f^{(4)}(c), \quad \left|E_n^S\right| \le \frac{(b-a)^5}{180n^4} M$

**POPPER16**

1. Which method will have the smallest error?

2. Which method will give the largest estimate for $\int_0^2 x^2\,dx$ with $n = 10$?

More Examples:

Use the table below to approximate $\int_0^2 f(x)\,dx$ with $n = 10$.

(a) using the trapezoid method

(b) using Simpson's rule.

| $x$ | $f(x)$ |
|---|---|
| 0 | 1.8 |
| 0.2 | 1.8 |
| 0.4 | 2.4 |
| 0.6 | 1.5 |
| 0.8 | 2.1 |
| 1 | 2.5 |
| 1.2 | 2.3 |
| 1.4 | 2.2 |
| 1.6 | 1.7 |
| 1.8 | 2.1 |
| 2 | 2.5 |

Estimate the error if $T_8$ is used to calculate $\int_0^5 \cos(3x)\,dx$

Estimate the error if $S_8$ is used to calculate $\int_0^5 \cos(3x)\,dx$

Find $n$ so that $T_n$ is guaranteed to approximate $\int_0^3 \cos(2x)\,dx$ to within 0.03

Find $n$ so that $S_n$ is guaranteed to approximate $\int_0^3 \cos(2x)\,dx$ to within 0.03

**POPPER16**

3. What comes next in the *sequence* 3, 6, 11, 18, 27, 38,…?

4. What is the formula for $a_n$ for the *sequence* 3, 6, 11, 18, 27, 38,…?

Sequences are LISTS of objects. The objects could be numbers or something else. The list in poppers 1 and 2 are sequences of numbers. Each number "has a place".

Formally, a sequence of numbers is a function from the positive integers (or natural numbers) to the real numbers:

$$f(n) = a_n, \ \ n \in \mathbb{N} \ (n = 1, 2, 3, \ldots)$$

One of the most important aspects (from our perspective) will be something called the "limit of a sequence".

Some facts:

- Sequences of numbers do not have to have a pattern or nice behavior.
- Most sequences that we deal with will have a pattern and “reasonably” nice behavior.
- The pattern will come from a *generating formula*.
- The nice behavior will come in the from of a *limiting behavior.*
- There are a variety of ways to denote a sequence: $a_n$, $\{ a_n \}$ or $\{ a_n \}^{\infty}_{n=1}$
- We will be concerned with infinite sequences.

Example: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$

What is the function associated with this sequence?

Example: $-1, 1, -1, 1, -1, 1, -1, 1, \ldots$

What is the function associated with this sequence?

Give the first 3 terms of each of the following sequences.

$$a_n = \frac{1}{n+2}$$

$$a_n = \frac{n}{1-2n}$$

$$a_n = \frac{(-1)^n}{n}$$

**Terms:**

Bounded sequence or set – The sequence or set fits inside an interval.

Upper bound – A number greater than or equal to all the elements of the sequence or set.

Least Upper Bound (LUB) – Smallest number greater than or equal to all the elements of the sequence or set.

Lower bound – A number less than or equal to all the elements of the sequence or set.

Greatest Lower Bound (GLB) – Largest number less than or equal to all the elements of the sequence or set.

Give several lower bounds for [–2, 3).

Give several upper bounds for [–2, 3).

Give the LUB and GLB of [–2, 3).

Give the LUB and GLB for $\{ x \mid x^2 < 4 \}$.

**5.** Give the LUB for [–3, 1).