Math 1432

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Class webpage:
http://www.math.uh.edu/~bekki/Math1432.html

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State whether the **sequence** converges as n → ∞.

1. $e^{\frac{-7}{n}}$.

2. $\dfrac{8\,ln\,n}{n}$

3. $\dfrac{5^{375n}}{n!}$

4. $$\frac{8^{n+1}}{9^{n-1}}$$

5. $$\int_0^n e^{-8x}\,dx$$

6. $$\int_{-n}^{n} \frac{8}{1+x^2}\,dx$$

<u>Sequences:</u> Let $\{ a_n \}$ be a sequence of real numbers.

Possibilities:

1) If $\lim_{n\to\infty} a_n = \infty$ then $\{ a_n \}$diverges to infinity.

2) If $\lim_{n\to\infty} a_n = -\infty$, then $\{ a_n \}$ diverges to negative infinity.

3) If $\lim_{n\to\infty} a_n = c$, a finite real number, then $\{ a_n \}$ converges to c.

4) If $\lim_{n\to\infty} a_n$ oscillates between two numbers, then $\{ a_n \}$ diverges by oscillation.

If a sequence has a finite limit as n approaches infinity, we say that the sequence **converges**.
If a sequence does not have a finite limit as n approaches infinity, then it **diverges**.

9.3 Infinite **Series**

Series vs. Sequence

First and most important – a sequence is a LIST and a series is a SUM.

Sequence: $\{a_n\} = a_1, a_2, a_3, \ldots, a_n, \ldots$

Series: $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$

Notation for series:

$\sum$ Sigma – means summation

$\sum_{k=0}^{n} a_k =$

$\sum_{n=0}^{\infty} a_n =$

Examples:

$$\sum_{n=1}^{\infty} \frac{1}{n} =$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} =$$

Properties:

$$\sum_{k=0}^{n} \alpha a_k = \alpha \sum_{k=0}^{n} a_k$$

$$\sum_{k=0}^{n} \left(a_k + b_k \right) = \sum_{k=0}^{n} a_k + \sum_{k=0}^{n} b_k$$

$$\sum_{k=0}^{m} a_k + \sum_{k=m+1}^{n} a_k = \sum_{k=0}^{n} a_k$$

The sum of a finite series is denoted by S_n where $S_n = \sum_{k=0}^{n} a_k$

For an *infinite* series, we are taking an infinite sequence $(a_0, a_1, a_2, ...)$ and adding all the terms together. But since there are an infinite amount of terms, doing this (literally) would be impossible so we will begin by looking at the *partial sums:*

$$S_0 = \sum_{k=0}^{0} a_k = a_0$$

$$S_1 = \sum_{k=0}^{1} a_k = a_0 + a_1$$

$$S_2 = \sum_{k=0}^{2} a_k = a_0 + a_1 + a_2$$

$$\vdots$$

$$S_n = \sum_{k=0}^{n} a_k = a_0 + a_1 + a_2 + \ldots + a_n$$

$$\vdots$$

If we determine that the sequence of these partial sums has a limit, then that limit would be the sum of the infinite series $\sum_{k=0}^{\infty} a_k$. In other words, $\sum_{k=0}^{\infty} a_k = \lim_{n\to\infty} \{S_n\}$

What is the sequence of partial sums for each?

$$\sum_{k=0}^{\infty} 1 =$$

$$\sum_{k=0}^{\infty} r =$$

$$\sum_{k=0}^{\infty} \frac{1}{2^k} =$$

If the sequence of partial sums converges to a finite limit L, we write $\sum_{k=0}^{\infty} a_k = L$ and say that the series $\sum_{k=0}^{\infty} a_k$ converges to L. We call L the sum of the series. If the sequence of partial sums diverges, we say that the series $\sum_{k=0}^{\infty} a_k$ diverges.

THEOREM: The *kth term* of a convergent series tends to 0; namely,

$$\text{if } \sum_{k=0}^{\infty} a_k \text{ converges, then } a_k \to 0 \text{ as } k \to \infty$$

THEOREM: BASIC DIVERGENCE TEST

$$\text{If } a_k \not\to 0 \text{ as } k \to \infty, \text{ then } \sum_{k=0}^{\infty} a_k \text{ diverges.}$$

****The Basic Divergence Test only proves <u>divergence</u>.****

More General Properties:

- If $\sum_{k=0}^{\infty} a_k$ converges and $\sum_{k=0}^{\infty} b_k$ converges,

  then $\sum_{k=0}^{\infty} \left(a_k + b_k \right)$ converges.

- If $\sum_{k=0}^{\infty} a_k$ converges, then $\sum_{k=0}^{\infty} \alpha a_k$ converges.

Ex. 1 Does $\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$ converge or diverge?

Telescoping Series:

A series such as $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\ldots$ is called a telescoping series because it collapses to one term or a few terms. If a series collapses to a finite sum, then it converges.

Ex. 2. Does the series 1 – 1 + 1 – 1 + 1 … converge or diverge?
How would we write the series using sigma notation?

Important examples:

$$\int_1^{\infty} \frac{dx}{x}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} =$$

$$\int_1^\infty \frac{dx}{x^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} =$$

Series (SUM)

$$\sum_{k=1}^{4} \frac{1}{k^2} =$$

$$\sum_{n=1}^{6} \frac{(-1)^n}{5n+1} =$$

$$\sum_{n=0}^{5} \left(\frac{1}{2} \right)^n =$$

$$\sum_{n=1}^{\infty} \left(\frac{-1}{2} \right)^n =$$

$$\sum_{n=1}^{\infty} (-1)^n =$$

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7. Which of the following diverge by the BDT?

a. $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n}$

b. $\sum_{n=1}^{\infty}\frac{1}{n}$

c. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n}$

d. $\sum_{n=1}^{\infty}\left(\frac{6}{11}\right)^{n}$