Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

So here is what we know so far:

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$

$\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Series – diverges

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ P-Series – converges if p>1, diverges otherwise

$\sum_{n=0}^{\infty} (r)^n$ Geometric – converges if $|r| < 1$ to $\frac{a_1}{1-r}$ and diverges if $|r| \geq 1$

Basic Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n > 0$

1. If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges

2. If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

Limit Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n \geq 0$

If you know $\sum_{n=1}^{\infty} b_n, b_n \geq 0$

1. If $\sum_{n=1}^{\infty} b_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ (L is any finite number),then $\sum_{n=1}^{\infty} a_n$ converges

2. If $\sum_{n=1}^{\infty} b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

The Integral Test:
If f is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then
$\sum_{n=1}^{\infty} a_n$ *and* $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

The Root Test:

Let $\sum a_k$ be a series with nonnegative terms. Suppose $(a_k)^{1/k} \to \rho$, then

1. $\sum a_k$ converges if $\rho < 1$
2. $\sum a_k$ diverges if $\rho > 1$
3. The test is inconclusive if $\rho = 1$

The Ratio Test:

Let $\sum a_k$ be a series with positive terms. Suppose $\frac{a_{k+1}}{a_k} \to \lambda$, then

1. $\sum a_k$ converges if $\lambda < 1$
2. $\sum a_k$ diverges if $\lambda > 1$
3. The test is inconclusive if $\lambda = 1$

Popper 25

1. Use the Ratio test to determine if the following are convergent or divergent (or if test is inconclusive).

$$\sum \frac{2^n}{n!}$$

2. $\sum_{n=0}^{\infty}\left(\frac{1-5^n}{6^n}\right)$

3. $\sum_{n=1}^{\infty}\frac{(n-1)!}{n!}$

4. $$\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{n^3}$$

5. $$\sum_{n=1}^{\infty} (-1)^n$$

6. $$\sum_{n=1}^{\infty} \frac{n^2}{n^5+4n+3}$$

9.6 Absolute Convergence and Alternating Series

An **alternating series** is a series whose terms alternate in sign. For example:

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots=\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{2^{n-1}}\right)$$

Alternating Series Test:

If an alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}(a_n)=a_1-a_2+a_3-a_4+\cdots,\quad a_n>0$

satisfies (*i*) $a_n \geq a_{n+1}$ for all *n* (non increasing) AND (*ii*) $\lim_{n\to\infty} a_n = 0$ then the series is convergent.

Examples:

1. $$\sum \frac{(-1)^n}{\sqrt{n^2 - 1}}$$

2. $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n^3 + 1}}{n+3}$$

3. $$\sum (-1)^n$$

If an alternating series converges, it can be classified as either absolutely convergent or conditionally convergent.

If the series $\sum |a_n|$ is convergent, then $\sum a_n$ is convergent. We say that $\sum a_n$ is **absolutely convergent**.

If the series $\sum a_n$ is convergent and the series $\sum |a_n|$ is divergent, we say that $\sum a_n$ is **conditionally convergent**.

Examples:

4. $\sum \frac{(-1)^n}{\sqrt{n^2 - 1}}$

5. $\sum \frac{\sin(\frac{\pi}{2}k)}{k\sqrt{k}}$

6. $\sum(-1)^n \frac{n}{\sqrt{n^2-1}}$

7. $\sum (-1)^k \left(\sqrt{k+1} - \sqrt{k}\right)$

If a convergent alternating series satisfies the condition $0 < a_{n+1} < a_n$, then the **remainder** R_N involved in approximating the sum S by S_N **is less in magnitude than the first neglected (truncated) term.** That is, $|R_N| = |S - S_N| \leq |a_{N+1}|$

Examples:

1. Approximate the sum of $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n!} \right)$ by its first six terms and find the error.

2. Find the smallest integer n so that s_n will approximate $\sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+2}$ to within 0.01

Popper 25

7. Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum \frac{\cos(\pi k)}{k\sqrt{k}}$$