Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Geometric Series Test.

Basic Divergence Test.

p-Series Test.

Integral Test.

Basic Comparison Test.

Limit Comparison Test.

Root Test

Ratio Test

Alternating Series Test for Convergence: $\sum_{n=1}^{\infty}(-1)^{n-1} b_n \quad b_n > 0$

$\sum_{n=1}^{\infty} a_n$ is **absolutely convergent** if $\sum_{n=1}^{\infty} |a_n|$ converges.

$\sum_{n=1}^{\infty} a_n$ is **conditionally convergent** if $\sum_{n=1}^{\infty} a_n$ converges

but $\sum_{n=1}^{\infty} |a_n|$ diverges.

(Note: a non-alternating series can never converge conditionally)

Popper 26

1. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2}$$

2. $$\sum_{n=1}^{\infty} \frac{2n+1}{5n^2+2n}$$

3. $$\sum_{n=1}^{\infty} \frac{3n+1}{5n^3+2n}$$

4. $$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

5. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

6. $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \arctan(n)}{n^2}$$

7. $$\sum_{n=1}^{\infty} \frac{n \cos(n\pi)}{2^n}$$

8. $$\sum_{n=1}^{\infty} \frac{n \cos(n\pi)}{n^2+1}$$

9. $$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

10. $$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$$

Notes for series "growth":

Let p(k) be a polynomial in k.

r^k for r > 1 grows much faster than p(k)

k! grows much faster than r^k, p(k)

k^k grows much faster than the others

Hence,

$$\sum \frac{p(k)}{r^k}, \quad \sum \frac{p(k)}{k!}, \quad \sum \frac{p(k)}{k^k}$$

$$\sum \frac{r^k}{k!}, \quad \sum \frac{r^k}{k^k}, \quad \sum \frac{k!}{k^k}$$

ALL converge rapidly.

Power Series:

Suppose that $f\left(x\right)=\dfrac{6}{1-x}$.

If you divide 1 – x into 6, you get a "polynomial" that continues forever.

$$P\left(x\right)=6+6x+6x^{2}+6x^{3}+6x^{4}+\ldots$$

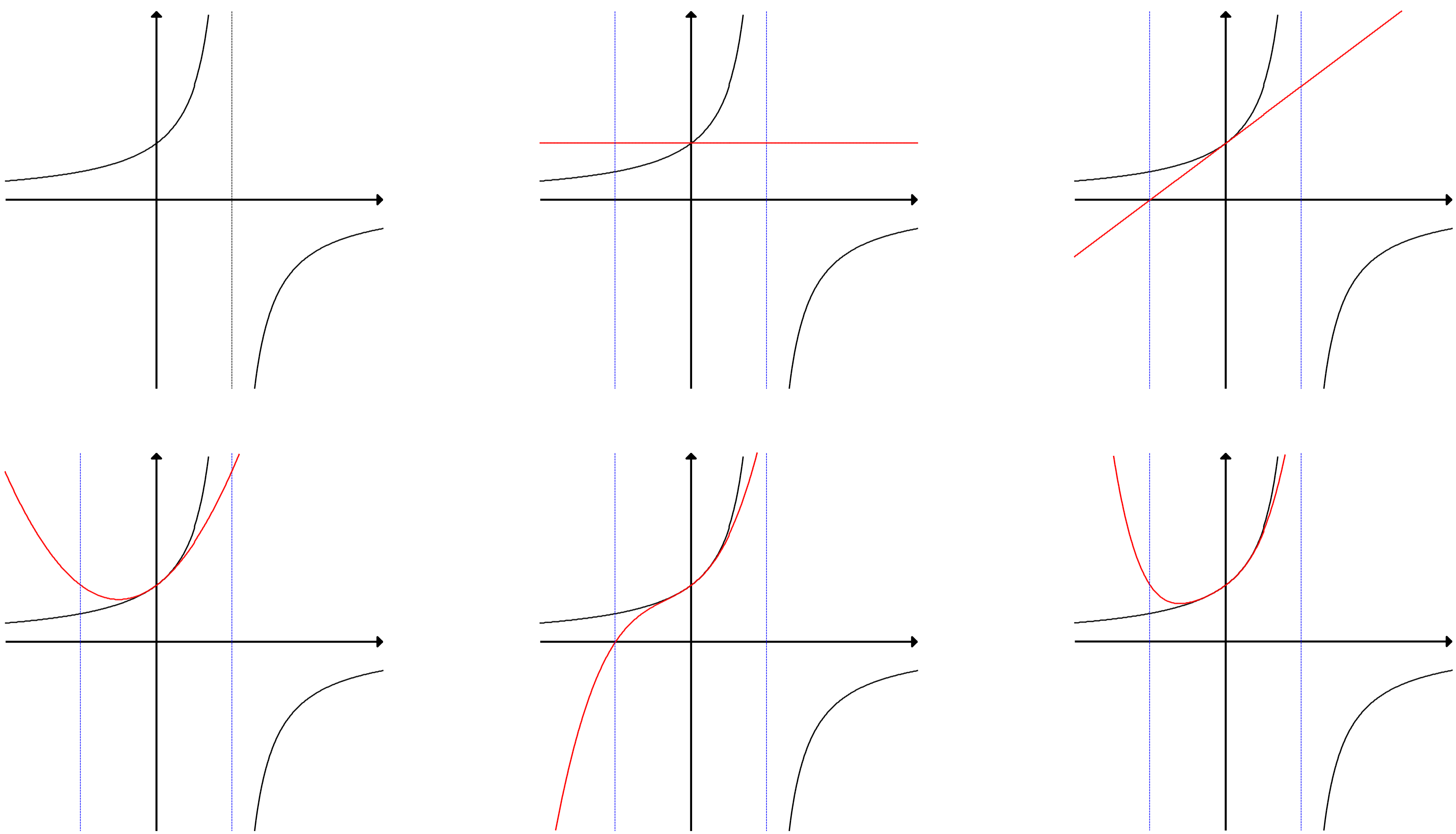
This result is a power series.

The word series indicates that there is an infinite number of terms.

The word power tells us that each term contains a power of x.

The series is also a geometric series, with $|r|=x$, so the series will converge for $|x|<1$.

By comparing the graphs of $f\left(x\right)=\dfrac{6}{1-x}$ and P(x) with more and more terms, you will see that between –1 and 1 (the interval of convergence), the two graphs converge.



A Power Series (centered at x=0) is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \ldots$$

where x is a variable and the c_n's are coefficients.

Note: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ when $|x|<1$

Using this, we can write functions in this form in sigma notation:

Ex: Write $\frac{x^2}{4-x^2}$ as its power series

For a **fixed** x, the series is a series of constants and we can check for convergence/divergence. The series may converge for some values of x and diverge for others.

The sum of the series is

$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \ldots + c_n x^n + \ldots$$

whose domain is the set of all x for which the series converges.

f (x) resembles a polynomial, but it has infinitely many terms.

Let $c_n = 1$ for all n, we get the geometric series, centered at $x = 0$,

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \ldots + x^n + \ldots$$

which converges if $|x| < 1$ and diverges if $|x| \geq 1$.

A Power Series (centered at $x=a$) is a series of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + c_3 (x-a)^3 + \ldots$$

For notation purposes, $(x - a)^0 = 1$ even when $x = a$.

When $x = a$, all the terms are 0 for $n \geq 1$, so the power series always converges when $x = a$.

Ex. For what values of x is the series convergent?

$$\sum_{n=0}^{\infty} n!x^n$$

For a given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ there are only 3 possibilities.

1. The series converges only when $x = a$.

2. The series converges for all x.

3. There is a positive number R such that the series converges if $|x - a| \leq R$ and diverges if $|x - a| > R$.

R is the radius of convergence.

The interval of convergence of a power series is the interval that consists of all values of x for which the series converges absolutely. Check endpoints (endpoints may converge absolutely or conditionally)!

Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$.

Find the radius of convergence and interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n2^n}.$$

Find the radius of convergence and interval of convergence for

$$\sum_{n=0}^{\infty} \left(\frac{x}{3} \right)^n .$$

Find the radius of convergence and interval of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}.$$

Find the radius of convergence and interval of convergence for

$$\sum_{n=0}^{\infty} n!(x-3)^n .$$

Power series are continuous functions.

A power series is continuous on its interval of convergence.

If a power series centered at $x = a$ has a radius of convergence $R > 0$, then the power series can be differentiated and integrated on $(a - R, a + R)$, and the new series will converge on $(a - R, a + R)$, and maybe at the endpoints.