Math 2311
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Office Hours: MW 11am to 12:45pm in 639 PGH
Online Thursdays 4-5:30pm
And by appointment

Class webpage: http://www.math.uh.edu/~bekki/Math2311.html
7. Determine if events $A$ and $B$ are independent.

\[ P(A) \cdot P(B) = P(A \cap B) \]

a. $P(A) = 0.9$, $P(B) = 0.3$, $P(A \cap B) = 0.27$

\[(0.9)(0.3) = 0.27 \quad \text{Yes} \]

b. $P(A) = 0.4$, $P(B) = 0.6$, $P(A \cap B) = 0.20$

\[(0.4)(0.6) = 0.24 \neq 0.20 \quad \text{not ind.} \]

\[ P(A \cap B) = 0 \]
#21 from text:
Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those “felony” students, 40% go on to college. Of the ones who do not face a disciplinary action, 60% go on to college.

\[ \text{F = felony, C = college, } P(F) = .3 \]

\[ \text{P}(C|F^c) = .6, \quad \text{P}(C|F) = .4 \]

a. What is the probability that a randomly selected student both faced a disciplinary action and went on to college?

\[ \text{P}(F \cap C) = \frac{\text{P}(C \cap F)}{\text{P}(F)} = \frac{\text{P}(C \cap F)}{.3} \]

b. What percent of the students from the high school go on to college?

\[ \text{P}(C) = \text{P}(C \cap F) + \text{P}(C \cap F^c) = .12 + .42 = .54 \]

\[ .4 = \frac{\text{P}(C \cap F)}{.3} \]

\[ .12 = \text{P}(C \cap F^c) \]

\[ \boxed{.54} \]

\[ .12 = \text{P}(C \cap F^c) \]

\[ .12 \neq .12 \]

\[ \text{not independent} \]
\[
P(C \cap F^c) = \frac{P(C \cap F^c)}{P(F^c)}
\]

\[
P(C|F^c) = \frac{.6}{.7} = \frac{x}{.7}
\]

\[
.42 = x
\]

\[
P(F) = .3
\]

\[
P(F^c) = .7
\]
P(A) = 0.73, P(B) = 0.44, P(A ∪ B) = 0.89

1. P(A ∩ B) =
   a. 0.3212
   b. 0.2800
   c. 0.3836
   d. 0.6364

2. P(A | B) = \[ \frac{P(A \cap B)}{P(B)} \]
   a. 0.3212
   b. 0.2800
   c. 0.3836
   d. 0.6364
Suppose you are playing poker with a standard deck of 52 cards:

How many 5 card hands are possible?

\[ \binom{52}{5} = 2,598,960 \]

How many ways can you get 4 kings in a hand?

\[ \binom{4}{4} \times \binom{48}{1} = 48 \]

How many ways can you have any 4 of a kind hand?

\[ 13 \times \binom{4}{4} \times \binom{48}{1} = 624 \]

What is the probability of getting 4 of a kind?

\[ P(4 \text{ of a kind}) = \frac{624}{2,598,960} \approx 0.0002 \]
How many ways can you have 3 kings and 2 fives?

\[ \begin{align*}
\text{KKK} & \quad 5 \\
4 \ choose \ 3 & \quad 4 \ choose \ 2 \\
\end{align*} \]

\[ 4 \cdot 6 = 24 \]

How many ways can you get a full house?

\[ \begin{align*}
& \quad 3 \ of \ a \ kind + 2 \ of \ a \ kind \\
13 \ (4 \ choose \ 3) \cdot 12 \ (4 \ choose \ 2) & = 3744 \\
\end{align*} \]

What is the probability of getting a full house?

\[ P(\text{full house}) = \frac{3744}{2598960} = .0014 \]
Problems from Quiz 2:

A researcher randomly selects 2 fish from among 10 fish in a tank and puts each of the 2 selected fish into different containers. How many ways can this be done?

\[
10 \text{ P}_2 = \frac{10!}{(10-2)!} = 90
\]

An experimenter is randomly sampling 4 objects in order from among 61 objects. What is the total number of samples in the sample space?

Since this says "in order", this is a Permutation. the answer is
\[
61 \text{ P}_4 = 12524520
\]

This is if it didn't say "in order"

\[
\binom{61}{4} \text{ Choose } (61,4) = 521855
\]

How many license plates can be made using 3 digits and 4 letters if repeated digits and letters are not allowed?

\[
10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \cdot 26
\]

repeats are allowed

\[
10 \text{ P}_3 \cdot 26 \text{ P}_4 = 258,336,000
\]

Perm.
De Morgan’s Law \((E \cap F)^c = E^c \cup F^c\) \((E \cup F)^c = E^c \cap F^c\)

Let \(A = \{2, 7\}\), \(B = \{7, 16, 22\}\), \(D = \{34\}\) and \(S = \text{sample space} = A \cup B \cup D\). Find \((A^c \cap B^c)^c\).

\[A^c = \{14, 22, 34\}\]
\[(A^c \cap B^c)^c = \{2, 7, 16, 22\}\]

Popper 02

3. Let \(A = \{2, 7\}\), \(B = \{7, 16, 22\}\), \(D = \{34\}\) and \(S = \text{sample space} = A \cup B \cup D\). Identify \(B^c \cup A\).

a) \(\{2, 7, 16, 22\}\)
b) \(\{2, 16, 22, 34\}\)
c) \(\{2, 7, 34\}\)
d) \(\{2, 34\}\)
e) \(\{2, 7\}\)

\[B^c \cap A = \{2\}\]
In a shipment of 71 vials, only 13 do not have hairline cracks. If you randomly select one vial from the shipment, what is the probability that it has a hairline crack?

\[ P(\text{crack}) = \frac{58}{71} \]

**Popper 02**

4. In a shipment of 62 vials, only 14 do not have hairline cracks. If you randomly select one vial from the shipment, what is the probability that it has a hairline crack?

a) \( \frac{1}{14} \)

b) \( \frac{24}{31} \)

c) \( \frac{7}{24} \)

d) \( \frac{7}{31} \)

e) \( \frac{1}{62} \)
In a shipment of 54 vials, only 16 do not have hairline cracks. If you randomly select 3 vials from the shipment, what is the probability that none of the 3 vials have hairline cracks?

\[
\frac{16 \binom{3}{3}}{54 \binom{3}{3}} = \frac{560}{24804}.
\]

exactly 2 have cracks:

\[
\frac{38 \binom{2}{2} \cdot 16 \binom{1}{1}}{54 \binom{3}{3}} = \frac{703 \cdot 16}{24804} \approx 0.453
\]

The probability that a randomly selected person has high blood pressure (the event H) is \( P(H) = 0.4 \) and the probability that a randomly selected person is a runner (the event R) is \( P(R) = 0.3 \). The probability that a randomly selected person has high blood pressure and is a runner is 0.2. Find the probability that a randomly selected person either has high blood pressure or is a runner or both.

\[
P(H \cup R) = P(H) + P(R) - P(H \cap R)
\]

\[
= 0.4 + 0.3 - 0.2
\]

\[
= 0.5
\]
5. The probability that a randomly selected person has high blood pressure (the event $H$) is $P(H) = 0.4$ and the probability that a randomly selected person is a runner (the event $R$) is $P(R) = 0.3$. The probability that a randomly selected person has high blood pressure and is a runner is 0.2. Find the probability that a randomly selected person has high blood pressure and is not a runner.

a) 0.5  
b) 0.2  
c) 0.7  
d) 0.6  
e) 0.4

Are events $H$ and $R$ independent? Mutually exclusive? No  
$P(H \cap R) \neq 0$

$P(H \cap R^c) = P(H) - P(H \cap R) = 0.4 - 0.2 = 0.2$
\[ P(H \cap O) = 0.08 \quad P(H) = 0.16 \quad P(O) = 0.26 \]

Hospital records show that 16% of all patients are admitted for heart disease, 26% are admitted for cancer (oncology) treatment, and 8% receive both coronary and oncology care. What is the probability that a randomly selected patient is admitted for coronary care, oncology, or both? (Note that heart disease is a coronary care issue.)

\[ P(H \cup O) = 0.16 + 0.26 - 0.08 \]

\[ = 0.34 \]

What is the probability that a randomly selected patient is admitted for something other than coronary care?

\[ P(H^c) = 0.84 \]

\[ = 1 - 0.16 \]
6. Among 9 electrical components exactly one is known not to function properly. If 3 components are randomly selected, find the probability that all selected components function properly.

(a) \( \frac{8C_3}{9C_3} \)

(b) \( \frac{8C_3}{9C_3} \)

(c) \( \frac{8}{9} \)

(d) \( \frac{5}{9} \)

(e) 1

What is the probability that exactly one does not function properly?

\[ \frac{C_1 \cdot 8C_2}{9C_3} = \frac{1}{3} \]

What is the probability that at least one does not function properly?

\[ \frac{1 \text{ or more}}{1 \text{ or } 2 \text{ or } 3} \]

\[ \frac{1}{3} + 0 + 0 = \frac{1}{3} \]
Section 3.1

A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.

A **discrete random variable** is one that can assume a countable number of possible values. A **continuous random variable** can assume any value in an interval on the number line.

A **probability distribution table of** $X$ **consists of all possible values of a discrete random variable with their corresponding probabilities.**

Example: Suppose a family has 3 children. Show all possible gender combinations:

$$\text{sex} = \{ \text{GGG, GGB, GBB, BBB, BGG, BGB, BGG, BBB} \}$$

Now suppose we want the probability distribution for the number of girls in the family.

$$\text{discrete RV} \implies X = \# \text{ of girls}$$

$$3 \text{ or } 2 \text{ or } 1 \text{ or } 0$$
Draw a probability distribution table for this example.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

$\sum P(X) = 1 \left( \frac{8}{8} \right)$

Find $P(X > 2)$

$P(X = 3) = \frac{1}{8}$

$P(X < 1) = \frac{1}{8}$

$P(1 < X \leq 3) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$
The mean, or expected value, of a random variable $X$ is found with the following formula:

$$
\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n
$$

What is the expected number of girls in the family above?

$$
E[X] = 0 \left( \frac{1}{8} \right) + 1 \left( \frac{3}{8} \right) + 2 \left( \frac{3}{8} \right) + 3 \left( \frac{1}{8} \right) = \frac{3}{2}
$$
The variance of a random variable $X$ can be found using the following:

$$\sigma_x^2 = \text{Var}[X] = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + \cdots + (x_n - \mu_x)^2 p_n$$

$$= \sum (x_i - \mu_x)^2 p_i$$

An alternate formula is:

$$\sigma_x^2 = \text{Var}[X] = E[X^2] - (E[X])^2$$

Find the standard deviation for the number of girls in the example above.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^2$</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>$p(x)$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

$$E[X^2] = 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 4\left(\frac{3}{8}\right) + 9\left(\frac{1}{8}\right) = 3$$

$$\text{VAR} = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$
Given the following sampling distribution:

<table>
<thead>
<tr>
<th>X</th>
<th>-18</th>
<th>-14</th>
<th>2</th>
<th>11</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>$\frac{1}{25}$</td>
<td>$\frac{3}{50}$</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{1}{100}$</td>
<td>___</td>
</tr>
</tbody>
</table>

7. What is $P(X=20)$?
   a. 16/100  b. 84/100  c. 53/100  d. none of these

8. What is $P(X>2)$?  
   a. 24/25  b. 9/10  c. 85/100  d. none of these

9 & 10 are A.