Math 2433
12021 - SR 117 - MWF 12-1

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14.6 Second Partial Derivatives

In the cases of higher order derivatives:

\[(f_x)_x = f_{xx} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial x^2}\]

\[(f_x)_y = f_{xy} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial y \partial x}\]

\[(f_y)_x = f_{yx} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial x \partial y}\]

\[(f_y)_y = f_{yy} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial y^2}\]
Example 1: Find all first and second partial derivatives of $f(x, y) = 2x^2 \cos(y) + 3y^2 \sin(x)$.

Solution:
Example 2: Find all first and second partial derivatives of
\[ f(x, y, z) = 4xe^y + 3ye^z + 2ze^x. \]

Solution:
Example 3: Find all first and second partial derivatives of 
\[ f(x, y, z) = x^2 y + y^2 z + z^2 x. \]
Solution:
1. Find $f_x$ if $f(x, y) = \frac{y}{x + y^2}$. 
2. Find $f_{xz}$ if $f(x, y, z) = 3xe^y + 2ye^z + 4ze^x$
Applications of Partial Derivatives

If plane is $y = y_0$, then slope of the tangent line is

And equation of tangent line will be

If plane is $x = x_0$, then slope of the tangent line is

And equation of tangent line will be
Applications of Partial Derivatives

Example: Let \( z = \frac{x^2}{y^2 - 1} \) and let \( C \) be the curve of intersection of the surface with the plane \( y = 2 \). Find equations for the line tangent to \( C \) at the point \( P(3, 2, 3) \).
Applications of Partial Derivatives

Example: Let \( z = \frac{x^2}{y^2 - 1} \) and let C be the curve of intersection of the surface with the plane \( x=3 \). Find equations for the line tangent to C at the point P(3, 2, 3).
A set is closed if it contains ALL of its boundary points.
A set is open if it contains NONE of its boundary points.

In the following examples, determine if the set is open, closed or neither:
1. $\{(x, y) \mid 2 < x < 5, 4 < y < 6\}$
2. \((x, y) \mid 2 \leq x \leq 5, \ 4 \leq y \leq 6\) 

3. \((x, y) \mid 2 \leq x \leq 5, \ 4 < y < 6\)
4. \( \{(x, y) \mid y \leq x^2\} \)

5. \( \{(x, y, z) \mid x^2 + y^2 \leq 1, \ z > 1\} \)
3. Is \( \{ (x, y, z) \mid x^2 + y^2 \leq 1, \ z \geq 1 \} \) open, closed or neither?