Math 3339

Section 21155
MW 1-2:30pm SR 117

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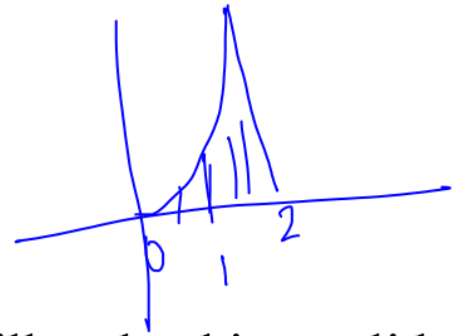
Office Hours:
M 11-12:30pm & T 3:00 – 4:00 pm
and by appointment

Test 2: 10 m/c and 4 f/r

m/c 4 or 5 pts ea. = 46 pts.

f/R 2 @ 15
2 @ 12 } 54 pts

Popper 15



1. Suppose $f(x) = \begin{cases} \dfrac{x^2}{k} & 0 \le x \le 2 \\ 0 & \textit{otherwise} \end{cases}$. What value of k will make this a valid pdf?

   a. 2
   b. 3/8
   c. 8/3
   d. 9
   e. none of these

$$\int_0^2 \frac{x^2}{k}\,dx = 1$$

$$\left.\frac{x^3}{3k}\right|_0^2 = \frac{8}{3k} = 1$$

2. For $f(x)$ above, find P(X<1).
   a. 1/3
   b. 1/9
   ~~c. 2/3~~
   d. 1/8
   e. none of these

$$\int_0^1 \frac{x^2}{\ }\,dx$$

General formula for continuous rv:

$$P(a < X \le b) = \int_a^b f(x)\,dx$$

$F'(x) = f(x)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(w)\,dw$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx \quad \text{and} \quad E[u(X)] = \int_{-\infty}^{\infty} u(x) f(x)\,dx$$

$$V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

HW 9

5. For healthy individuals the level of prothrombin in the blood is approximately normally distributed with mean 20 mg/100 mL and standard deviation 4 mg/100 mL. Low levels indicate low clotting ability. In studying the effect of gallstones on prothrombin, the level of each patient in a sample is measured to see if there is a deficiency. Let μ be the true average level of prothrombin for gallstone patients.

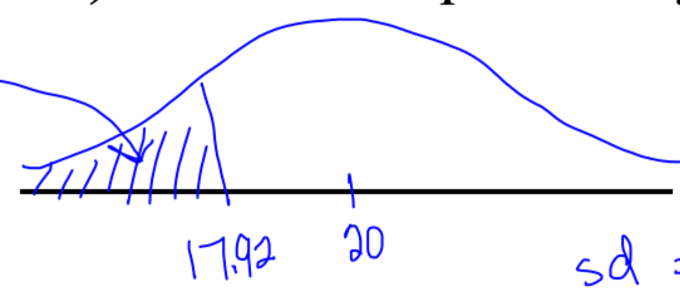
a. What are the appropriate null and alternative hypotheses?

$H_0: \mu \leq 20$

$H_a: \mu < 20$

b. Let $\bar{x}$ denote the sample average level of prothrombin in a sample of $n = 20$ randomly selected gallstone patients. Consider the test procedure with test statistic $\bar{x}$ and rejection region $\bar{x} \leq 17.92$. What is the probability distribution of the test statistic when H_0 is true (i.e. determine center, spread, and shape of $\bar{x}$)? What is the probability of a type I error for this test procedure?

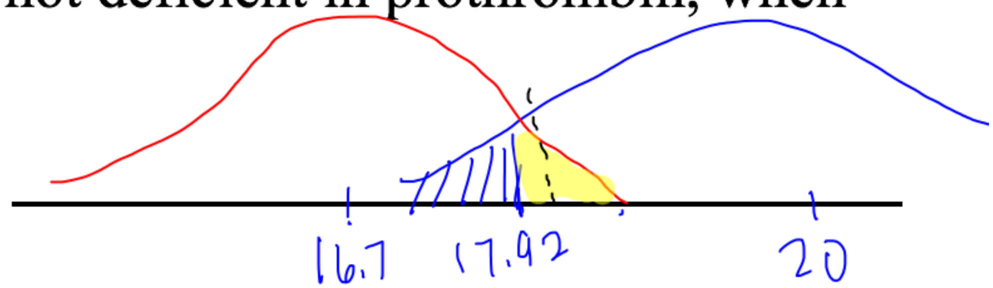
pnorm$(17.92, 20, 4/\sqrt{20})$



sd $= 4 = b$

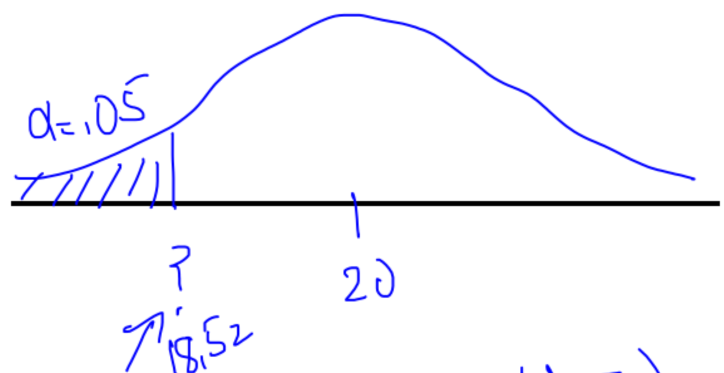
c. What is the probability distribution of the test statistic, $\bar{x}$ when μ = 16.7? Using the test procedure of part (b), what is the probability that gallstone patients will be judged not deficient in prothrombin, when in fact μ = 16.7 (a type II error)?

~Normal, $\mu = 16.7$, $\sigma = 4$



$P(\bar{X} > 17.92 \mid \mu = 16.7) = 1 - \text{pnorm}(17.92, 16.7, 4/\sqrt{20})$

d. How would you change the test procedure of part (b) to obtain a test with significance level 0.05? What impact would this change have on the error probability of part (c)?

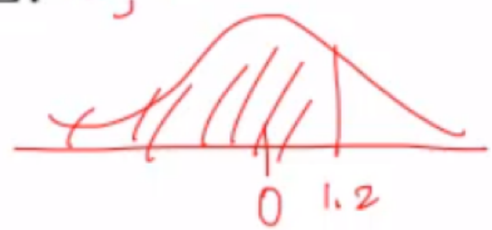


qnorm(.05, 20, $4/\sqrt{20}$)

9. Recall Z is the standard normal random variable.

a. What is the mean and standard deviation for Z? 0, 1

b. Sketch the distribution



c. Find $P(Z < 1.2)$

d. Find $P(Z < -1.64)$

e. Find $P(Z > -1.39)$

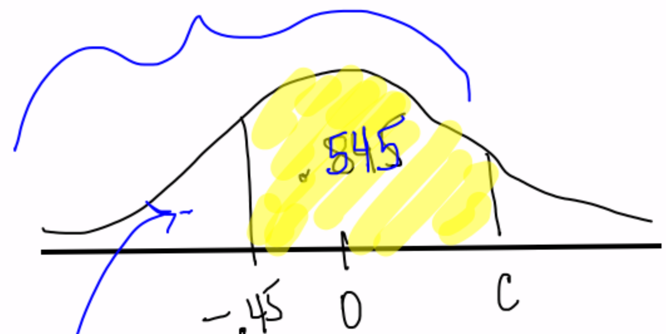
f. Find $P(-0.45 < Z < c) = 0.\cancel{8}545$ different on the key

$.326 + .545 = .871$

g. Find c such that $P(Z < c) = 0.845$

h. Find c such that $P(Z > c) = 0.845$

i. Find c such that $P(-c < Z < c) = 0.845$



ϕ

pnorm(-.45) = .326

c = qnorm(.871) = 1.13

HW 10

1. Approximately 60% of all part-time college students in the United States are female. (In other words, the population proportion of females among part-time college students is p = 0.6.)

   a. What would you expect to see in terms of the behavior of a sample proportion of females ($\hat{p}$) if random samples of size 100 were taken from the population of all part-time college students? That is, give the distribution (shape, center, and spread) of the sample proportion of part-time students that are females.

   b. If a random sample of size 100 were taken, what is the probability that less than 50% are females?

   c. If a random sample of size 100 were taken, there is a 95% chance that the sample proportion ($\hat{p}$) falls between what two values? find CI

$.6 \pm$

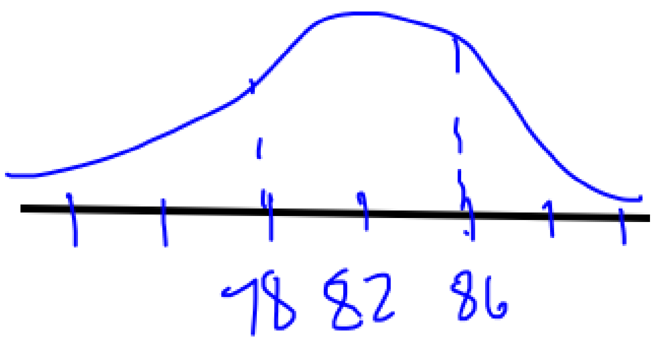
$$z = \frac{.5 - .6}{\sqrt{\frac{.6(.4)}{100}}} = \_\_\_\_ \qquad p(z < \_\_\_)$$

8. Let X be a normal random variable with $\mu = 82$ and $\sigma = 4$.

   a. Sketch the distribution
   b. According to the Empirical Rule, the middle 68% of the data falls between what values?
   c. Find $P(X < 83)$.
   d. Find $P(X > 79)$.
   e. Find $P(73 < X < 84)$
   f. Find x such that $P(X < x) = .97725$

use pnorm(_ , 82, 4)

qnorm(.97725, 82, 4)



17. A study of the ability of individuals to walk in a straight line reported the accompanying data on cadence (strides per second for a sample of n = 20 randomly selected men).

.95	.85	.92	.95	.93	.86	1.00	.92	.85	.81
.78	.93	.93	1.05	.93	1.06	1.06	.96	.81	.96

a. Find a 99% confidence interval for the mean cadence of the population.

b. Test the hypothesis that the mean cadence for the population is less than 0.97 at the 5% significance level.

I did this in R:

```
> walk=c(.95,.85,.92,.95,.93,.86,1,.92,.85,.81,.78,.93,.93,1.05,.93,1.06,1.06,.96,.81,
.96)
```

mean(walk) + c(-1,1) * qt(.995,19) * sd(walk)/sqrt(20)

b. $H_0: \mu = .97$

$H_a: \mu < .97$

$$t = \frac{\text{mean(walk)} - .97}{\text{sd(walk)/sqrt(20)}} =$$

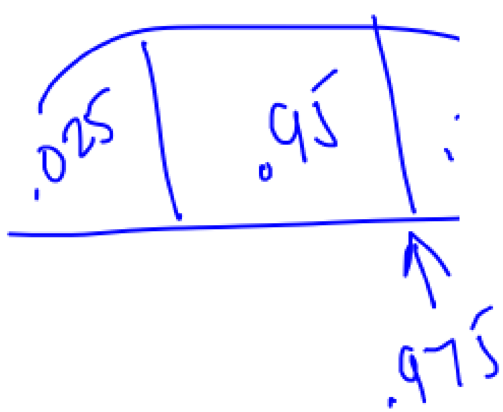
pvalue pt(, 19)

24. A manufacturer of car batteries claims that his batteries will last, on average, 3 years with a variance of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, construct a 95% confidence interval for the variance σ^2 and decide if the manufacturer's claim that $\sigma^2 = 1$ is valid. Assume the population of battery lives to be approximately normally distributed.

data = c(1.9, 2.4, 3.0, 3.5, 4.2)

(5-1)*var(data)/qchisq(.025,4)

(5-1)*var(data)/qchisq(.975,4)



In R-studio:

```
> batteries = c(1.9,2.4,3,3.5,4.2)
> lcl=4*var(batteries)/qchisq(1.95/2,4)
> ucl=4*var(batteries)/qchisq(0.05/2,4)
> c(lcl,ucl)
[1] 0.2925528 6.7297174
```

The confidence interval for the variance is: (0.2926, 6.7297).

Since $\sigma^2 = 1$ is inside the interval, the manufacturer's claim is valid.

Question 17

You did not answer the question.

$n = 100$

A simple random sample of 100 8th graders at a large suburban middle school indicated that 84% of them are involved with some type of after school activity. Find the 90% confidence interval that estimates the proportion of them that are involved in an after school activity.

a) (0.700, 0.900)

b) (0.780, 0.700)

c) (0.780, 0.900)

d) (0.830, 0.835)

e) (0.680, 0.850)

f) None of the above

$$.84 \pm \text{qnorm}(.95) * \sqrt{\frac{.84(.16)}{100}}$$

$$\frac{1+.9}{2}$$

poppers 3-7 = A