Math 3339

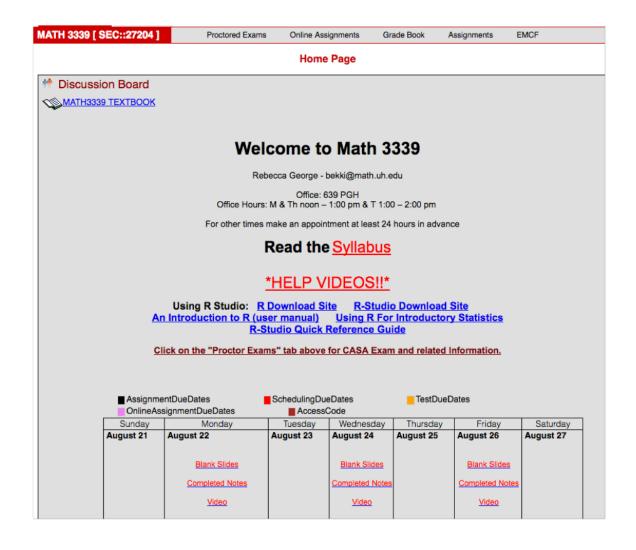
Section 27204 MWF 10-11:00am AAAud 2

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Office Hours: M & Th noon – 1:00 pm & T 1:00 – 2:00 pm and by appointment

All class information is found on CASA

https://www.casa.uh.edu/



The **Conditional Probability** of an event A given that event B has occurred is given by

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \quad \text{where } P(B) \neq 0$$

Prob (A given B)

Ex: Suppose that of all individuals buying a certain digital camera, 60% include an optional memory card in their purchase, 40% include an extra battery, and 30% include both a card and battery. Consider randomly selecting a buyer and let

 $A = \{\text{memory card purchased}\}\$ $B = \{\text{battery purchased}\}\$

Given that the selected individual purchased an extra battery, determine the probability that a memory card was also purchased.

$$P(A) = .6 \qquad P(B) = .4 \qquad P(A \cap B) = .3$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.3}{.4} = .75$$

Ex: A news magazine publishes three columns entitled "Art" (A), "Books" (B), and "Cinema" (C). Reading habits of a randomly selected reader with respect to these columns are

Read regularly A B C A
$$\cap$$
 B A \cap C B \cap C A \cap B \cap C A \cap B \cap C A \cap B \cap C B \cap

Determine the probability that the reader reads the art column if it is known that he read the books column.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.08}{.23} = \frac{.8}{.23}$$

Determine the probability that he reads the art column given that he has read at least one of the other columns.

$$P(A \mid (BUC)) = \frac{P(A \cap (BUC))}{P(BUC)} = \frac{.12}{.47} = \frac{12}{.47}$$

Determine the probability the reader will read the art column given that he will read at least one column.

$$P(A \mid (AUBUC)) = \frac{P(A \cap (AUBUC))}{P(AUBUC)}$$

$$= \frac{P(A)}{P(AUBUC)} = \frac{.14}{.49} = \frac{.2}{.49}$$

$$P(A|B) = \frac{8}{23}$$

 $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{.09}{.37} = \frac{9}{37}$

$$\frac{8}{23} + \frac{9}{37} =$$

$$P(AVB) = P(A) + P(B) - P(AVB)$$

 $P(AVB) = P(A) + P(B) - P(AVB)$

The Law of Total Probability Theorem:

Let $A_1, A_2, ..., A_k$ be mutually exclusive and exhaustive events. Then for any event B,

$$P(B) = P(B | A_1)P(A_1) + P(B | A_2)P(A_2) + ... + P(B | A_k)P(A_k)$$

$$= \sum_{i=1}^{k} P(B | A_i)P(A_i)$$

$$P(E|F) = \frac{P(EnF)}{P(F)}$$

$$P(E|F) \cdot P(F) = P(EnF)$$

Ex: An individual has 3 different email accounts. Most of her messages, in fact 70%, come into account #1, whereas 20% come into account #2, and the remaining 10% come into account #3. Of the messages into account #1, only 1% are spam, whereas the corresponding percentages for accounts #2 and #3 are 2% and 5% respectively. What is the probability that a randomly selected message is spam?

randomly selected message is spam?

$$P(A_1) = .7$$
 $P(A_2) = .2$
 $P(A_3) = .1$
 $P(S | A_1) = .01$
 $P(S | A_2) = .02$
 $P(S | A_3) = .05$
 $P(S) = P(S | A_1) + P(S | A_2) + P(S | A_3)$
 $= P(S | A_1) \cdot P(A_1) + P(S | A_2) \cdot P(A_2) + P(S | A_3) P(A_3)$
 $= .01 (.7) + .02 (.2) + .05 (.1)$
 $= .01 (.7)$

$$P(S|A_1) = .007 = P(S|A_1)$$

$$P(A_1) = .007 = P$$

Bayes' Theorem

If a random experiment can result in k mutually exclusive and exhaustive outcomes $A_1, A_2, A_3, ...A_k$, then for an event B

Case:
$$2 A$$
's $P(A_1 | B) = \frac{P(B | A_1)P(A_1)}{P(B)} = \frac{P(B | A_1)P(A_1)}{\sum_{i} P(B | A_i)P(A_i)}$

Case: $2 A$'s $P(A_1 | B) = \frac{P(B | A_1)P(A_1)}{P(B | A_1)P(A_1)} + P(B | A_2)P(A_2)$
 $P(A_1 | B) = \frac{P(B | A_1)P(A_1)}{P(B | A_1)P(A_1)} + P(B | A_2)P(A_2)$
 $P(A_1 | B) = \frac{P(B | A_1)P(A_1)}{P(B | A_1)P(A_1)} + P(B | A_2)P(A_2)$

Ex: Suppose that we know that 5 percent of the population have a certain disease. Suppose also, that no test for the disease is 100% accurate. A particular screening for the disease is proposed. In tests it is determined that this screening will be positive for a person without the disease 2% of the time, and the test will be negative for a person with the disease 8% of the time.

Determine the probability that a person who has tested positive for the

disease does not have the disease.

$$\frac{P(ND) \text{ sease } + +}{P(D' | T)} = \frac{P(D' \cap T)}{P(T)}$$

$$= \frac{(.95)(.02)}{(.05)(.92) + (.95)(.02)}$$

$$= .3923$$

