

# Math 3339

Section 27204

MWF 10-11:00am AAAud 2

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Office Hours:

M & Th noon – 1:00 pm & T 1:00 – 2:00 pm  
and by appointment



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# All class information is found on CASA

<https://www.casa.uh.edu/>

**MATH 3339 [ SEC::27204 ]**Proctored ExamsOnline AssignmentsGrade BookAssignmentsEMCF

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## Welcome to Math 3339

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For other times make an appointment at least 24 hours in advance

**Read the [Syllabus](#)**

**[\\*HELP VIDEOS!!\\*](#)**

Using R Studio: [R Download Site](#) [R-Studio Download Site](#)  
[An Introduction to R \(user manual\)](#) [Using R For Introductory Statistics](#)  
[R-Studio Quick Reference Guide](#)

Click on the "Proctor Exams" tab above for CASA Exam and related Information.

■ AssignmentDueDates

■ SchedulingDueDates

■ TestDueDates

■ OnlineAssignmentDueDates

■ AccessCode

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
August 21	August 22	August 23	August 24	August 25	August 26	August 27
	<a href="#">Blank Slides</a>		<a href="#">Blank Slides</a>		<a href="#">Blank Slides</a>	
	<a href="#">Completed Notes</a>		<a href="#">Completed Notes</a>		<a href="#">Completed Notes</a>	
	<a href="#">Video</a>		<a href="#">Video</a>		<a href="#">Video</a>	

Buy your popper (bubbling) forms and Course Access Code from the **Bookstore** by **9/6**. Take your class and section number to get the correct bubbling forms.

Print off notes (4 per page unless you write large) before class to make note-taking easier.

Some review:

A red 6-sided die and a green 6-sided die are thrown simultaneously. The outcomes of this experiment are equally likely. What is the probability that at least one of the dice lands with a 6 on its upper face?

$$6 \cdot 6 = 36 \leftarrow \# \text{ in sample space}$$

	green					
	1	2	3	4	5	6
red 1						
2	(2,1)					
3						
4						
5						
6						

$$\frac{11}{36}$$

$$P(\bar{6}R \cup 6\bar{6})$$

$$= P(6R) + P(6\bar{6}) - P(6R \cap 6\bar{6})$$

$$\frac{6}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{11}{36}$$

A hand of 5-card draw poker is a simple random sample from the standard deck of 52 cards. What is the probability that a 5-card draw hand contains the Jack of Spades? # Sample:  $52C_5 = \text{choose}(52, 5) = 2598960$

$$P(J_{Sp}) = \frac{{}^1C_1 \cdot {}^{51}C_4}{52C_5} = \frac{\text{choose}(1, 1) * \text{choose}(51, 4)}{\text{choose}(52, 5)} \approx .09615$$

What is the probability that a 5-card draw hand contains the Jack of Spades and the Ace of Hearts?

$$P(J_{Sp} + A_{H}) = \frac{{}^1C_1 \cdot {}^1C_1 \cdot {}^{50}C_3}{52C_5} = .00754$$

The **Conditional Probability** of an event  $A$  given that event  $B$  has occurred is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{where } P(B) \neq 0$$

↑ known  
given

$$P(A \cap B) = P(A|B) \cdot P(B)$$

## Bayes' Theorem

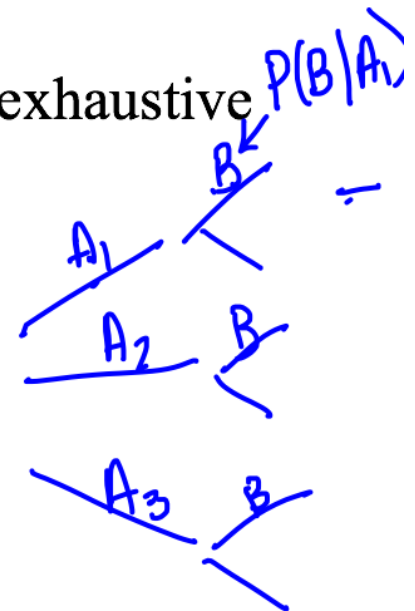
If a random experiment can result in  $k$  mutually exclusive and exhaustive outcomes  $A_1, A_2, A_3, \dots, A_k$ , then for an event  $B$

$$P(A_j | B) = \frac{P(B | A_j) P(A_j)}{P(B)} = \frac{P(B | A_j) P(A_j)}{\sum_i P(B | A_i) P(A_i)}$$

↑

$$P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) \dots$$

$$P(B|A_1) \cdot P(A_1) + P(B|A_2) P(A_2) + \dots$$



Ex: Items in your inventory are produced at three different plants, 50% from plant 1, 30% from plant 2, and 20% from plant 3. In addition, each plant produces at different levels of quality. Plant 1 produces 5% defectives, plant 2 produces 7% defectives, and plant 3 produces 8% defectives.

If an item from your inventory is found to be defective, <sup>given</sup> what is the probability that it was produced in plant 1?  $P(P_1 | D) = \frac{P(P_1 \cap D)}{P(D)}$

$$P(D \cap P_1) = P(D | P_1) \cdot P(P_1) = (.05)(.5) = .025$$

$$P(D \cap P_2) = P(D | P_2) \cdot P(P_2) = (.07)(.3) = .021$$

$$P(D \cap P_3) = P(D | P_3) \cdot P(P_3) = (.08)(.2) = .016$$

$$P(P_1 | D) = \frac{.025}{.062} \approx .403$$

$$P(D) = .062$$

$$\begin{array}{c}
 P(P_1) \\
 \hline
 .5 \\
 P_1
 \end{array}
 \quad
 \begin{array}{c}
 P(D|P_1) \\
 \hline
 .05 \quad D \\
 \hline
 .95 \quad D'
 \end{array}
 \rightarrow (.05)(.5) = .025 = P(P_1 \cap D)$$

$$\begin{array}{c}
 .3 \\
 \hline
 P_2
 \end{array}
 \quad
 \begin{array}{c}
 .07 \quad D \\
 \hline
 .93 \quad D'
 \end{array}
 \rightarrow (.07)(.3) = .021$$

$$\begin{array}{c}
 .20 \\
 \hline
 P_3
 \end{array}
 \quad
 \begin{array}{c}
 P(D|P_3) \\
 \hline
 .08 \quad D \\
 \hline
 .92 \quad D'
 \end{array}
 \rightarrow (.08)(.2) = .016$$


---


$$P(D) = .062$$



Mutually Exclusive (disjoint)  $A \cap B = \emptyset$   
 $P(A \cap B) = 0$

## Independence

Independence:

Two events are called **independent** if  $P(A \cap B) = P(A)P(B)$ .

In this case, what are the conditional probabilities?

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$\Rightarrow P(A|B) = P(A)$$

Two events that are not independent are called *dependent* events.

Ex: If we toss a nickel and dime at the same time, their outcomes are independent. Determine the probability that the nickel is heads and the dime is tails.

$$P(NH) = \frac{1}{2} \quad P(DT) = \frac{1}{2}$$

$$P(NH \cap DT) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Ex: Suppose that a machine has a vital component that fails with probability 0.01. In order to improve the probability that the machine runs properly, a second and third copy of this component is installed to run in parallel with the first. What is the probability that the machine runs properly?

$$\begin{aligned} P(\text{runs}) &= 1 - P(\text{all 3 fails}) = \leftarrow \text{at least one runs} \\ &= 1 - (.01)(.01)(.01) \\ &= .999999 \end{aligned}$$

Ex: Consider a gas station with six pumps numbered 1,2,...,6, and let  $E_i$  denote the simple event that a randomly selected customer uses pump  $i$  ( $i = 1,2,\dots,6$ ). Suppose that

$$P(E_1) = P(E_6) = 0.10 \quad P(E_2) = P(E_5) = 0.15 \quad P(E_3) = P(E_4) = 0.25$$

Define events  $A$ ,  $B$ , and  $C$  by

$$A = \{E_2, E_4, E_6\}, \quad B = \{E_1, E_2, E_3\}, \quad C = \{E_2, E_3, E_4, E_5\}.$$

Which events are independent? dependent?

$$P(A) = .15 + .25 + .10 = .5$$

$$P(B) = .1 + .15 + .25 = .5$$

$$P(C) = .15 + .25 + .25 + .15 = .8$$

---


$$A \text{ \& } B: \quad P(A) \cdot P(B) = P(A \cap B)?$$

$$\text{Dependent} \quad .25 \neq P(E_2) = .15$$

$$B \text{ \& } C: \quad P(B) \cdot P(C) = P(B \cap C)$$

$$\text{Independ.} \quad .5 \cdot .8 \quad P(E_2, E_3)$$

$$.4 \quad .15 + .25 = .4$$

$$\left\{ \begin{array}{ll} A \text{ \& } C: & P(A) \cdot P(C) \quad P(A \cap C) \\ & (.5)(.8) \quad P(E_2, E_4) \\ & .4 = .15 + .25 \\ & \text{Independent} \end{array} \right.$$

## Some R commands

Entering a list:

```
>x=c(1,2,3)
```

or

```
>x<-c(1,2,3)
```

```
> x
```

```
[1] 1 2 3
```

```
>y=c(4,5,6)
```

```
> rbind(x,y)
```

```
  [,1] [,2] [,3]
```

```
x    1    2    3
```

```
y    4    5    6
```

```
> cbind(x,y)
```

```
  x y
```

```
[1,] 1 4
```

```
[2,] 2 5
```

```
[3,] 3 6
```

```
> a[2,2]
```

```
y
```

```
5
```

```
> a[1,3]
```

```
x
```

```
3
```

```
> sum(a)
```

```
[1] 21
```

```
> colSums(a)
[1] 5 7 9
> rowSums(a)
x y
6 15
```

Let's put this in a table:

	Cash	Credit	Debit
<\$20	.09	.03	.04
\$20-\$100	.05	.21	.18
>\$100	.03	.23	.14

*from 3.6 #1*

What proportion of purchases are paid for using debit?

0.36

Given that a purchase is for less than \$20, what is the probability that it is paid for by cash?

$$P(\text{cash} | < \$20) = \frac{P(\text{cash} \cap < \$20)}{P(< \$20)}$$

> sum(tbl[1,])  
[1] 0.16

$$= \frac{.09}{.16} = \frac{9}{16}$$

> tbl[1,1]/sum(tbl[1,])  
cash  
0.5625

Are payment by cash and amount < \$20 independent events?

No

$$P(\text{cash}) \cdot P(< \$20) = P(\text{cash} \cap < \$20)$$

$$(\cancel{.14}, .17) (.16) \neq .09$$