Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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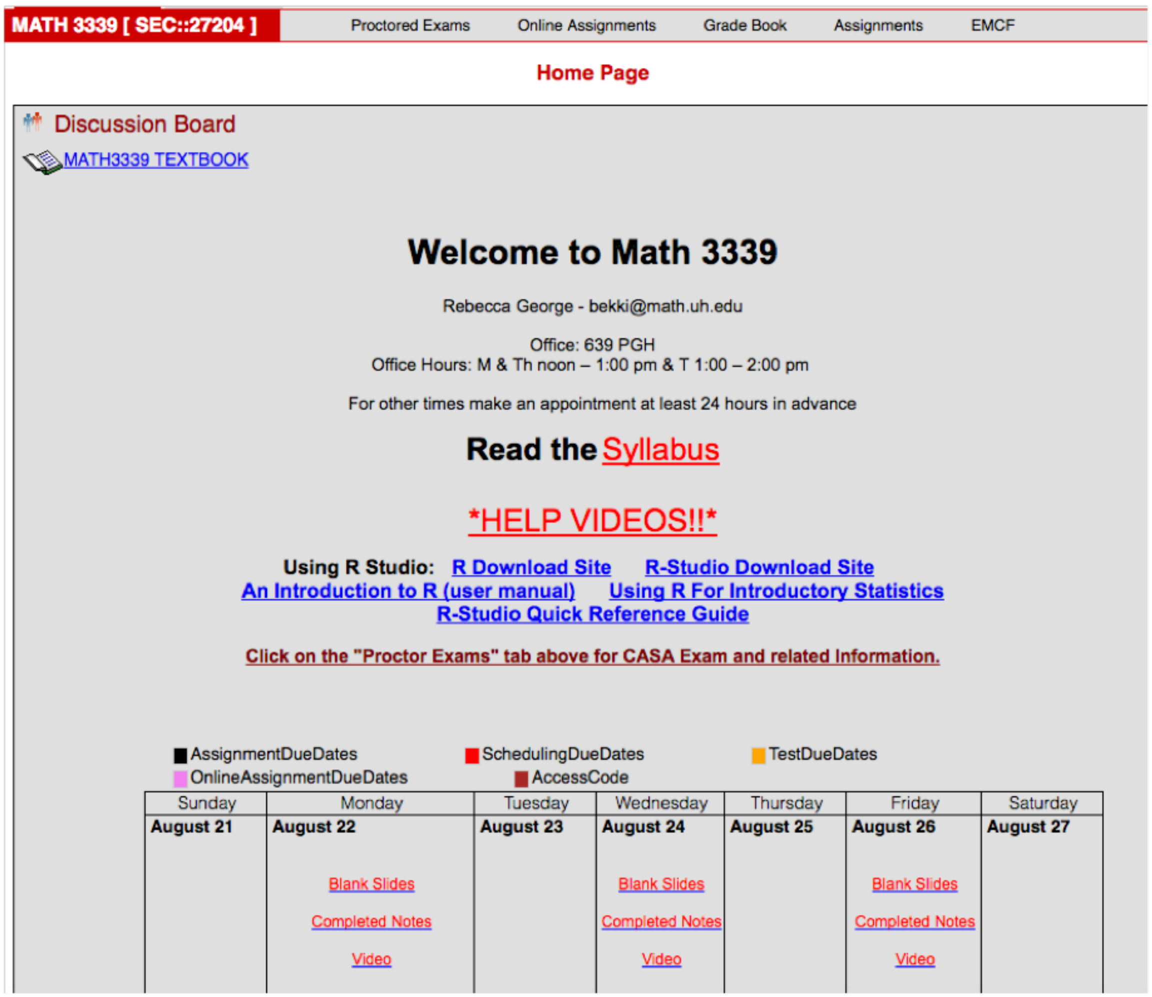
Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

All class information is found on CASA

https://www.casa.uh.edu/



Buy your popper (bubbling) forms and Course Access Code from the **Bookstore** by **9/6**. Take your class and section number to get the correct bubbling forms.

Print off notes (4 per page unless you write large) before class to make note-taking easier.

Some review:

A red 6-sided die and a green 6-sided die are thrown simultaneously. The outcomes of this experiment are equally likely. What is the probability that at least one of the dice lands with a 6 on its upper face?

$6 \cdot 6 = 36$ ← # in sample space

Red \ Green	1	2	3	4	5	6
1						
2	(2,1)					
3						
4						
5						
6						

$$\frac{11}{36}$$

$$P(6R \cup 6G)$$

$$= P(6R) + P(6G) - P(6R \cap 6G)$$

$$\frac{6}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{11}{36}$$

A hand of 5-card draw poker is a simple random sample from the standard deck of 52 cards. What is the probability that a 5-card draw hand contains the Jack of Spades? # Sample: ${}_{52}C_5$ = choose(52,5) = 2598960

$$P(J\spadesuit) = \frac{{}_1C_1 \cdot {}_{51}C_4}{{}_{52}C_5} = \text{choose}(1,1) * \text{choose}(51,4) / \text{choose}(52,5)$$

$$\approx .09615$$

What is the probability that a 5-card draw hand contains the Jack of Spades and the Ace of Hearts?

$$P(J\spadesuit + A\heartsuit) = \frac{{}_1C_1 \cdot {}_1C_1 \cdot {}_{50}C_3}{{}_{52}C_5} = .00754$$

The **Conditional Probability** of an event A given that event B has occurred is given by

know

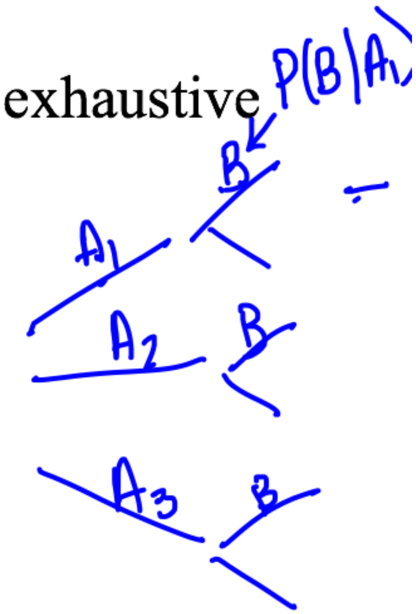
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{where } P(B) \neq 0$$

given

$$P(A \cap B) = P(A \mid B) \cdot P(B)$$

Bayes' Theorem

If a random experiment can result in k mutually exclusive and exhaustive outcomes $A_1, A_2, A_3, \ldots A_k$, then for an event B



$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{P(B)} = \frac{P(B \mid A_j)P(A_j)}{\sum_i P(B \mid A_i)P(A_i)}$$

$$P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) \cdots$$

$$P(B \mid A_1) \cdot P(A_1) + P(B \mid A_2)P(A_2) + \cdots$$

Ex: Items in your inventory are produced at three different plants, 50% from plant 1, 30% from plant 2, and 20% from plant 3. In addition, each plant produces at different levels of quality. Plant 1 produces 5% defectives, plant 2 produces 7% defectives, and plant 3 produces 8% defectives.

If an item from your inventory is found to be defective, what is the probability that is was produced in plant 1?

given

$$P(P1 \mid D) = \frac{P(P1 \cap D)}{P(D)}$$

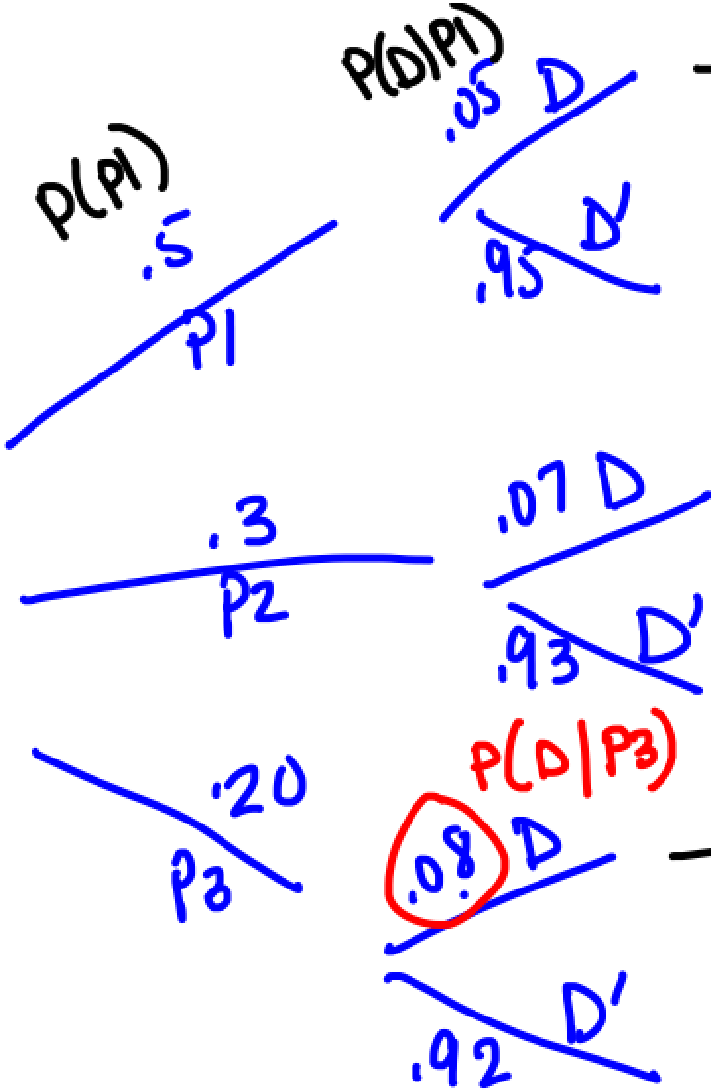
$$P(D \cap P1) = P(D \mid P1) \cdot P(P1) = (.05)(.5) = \boxed{.025}$$

$$P(D \cap P2) = P(D \mid P2) \cdot P(P2) = (.07)(.3) = .021$$

$$P(D \cap P3) = P(D \mid P3) \cdot P(P3) = (.08)(.2) = .016$$

$$P(D) = \boxed{.062}$$

$$P(P1 \mid D) = \frac{.025}{.062} \approx .403$$



$\longrightarrow (.05)(.5) = .025 = P(P1 \cap D)$

$\longrightarrow (.07)(.3) = .021$

$\longrightarrow (.08)(.2) = .016$

$P(D) = .062$

Mutually Exclusive (disjoint) $A \cap B = \emptyset$

$P(A \cap B) = 0$

Independence

Independence:

Two events are called *independent* if $\boxed{P(A \cap B) = P(A)P(B)}$.

In this case, what are the conditional probabilities?

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$\Rightarrow P(A|B) = P(A)$$

Two events that are not independent are called *dependent* events.

Ex: If we toss a nickel and dime at the same time, their outcomes are independent. Determine the probability that the nickel is heads and the dime is tails.

$$P(NH) = \frac{1}{2} \quad P(DT) = \frac{1}{2}$$

$$P(NH \cap DT) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Ex: Suppose that a machine has a vital component that fails with probability 0.01. In order to improve the probability that the machine runs properly, and second and third copy of this component is installed to run in parallel with the first. What is the probability that the machine runs properly?

$$P(\text{runs}) = 1 - P(\text{all 3 fails}) = \leftarrow \text{at least one runs}$$

$$= 1 - (.01)(.01)(.01)$$

$$= .999999$$

Ex: Consider a gas station with six pumps numbered 1,2,…,6, and let E_i denote the simple event that a randomly selected customer uses pump i (i = 1,2,…,6). Suppose that

$$P(E_1)=P(E_6)=0.10 \quad P(E_2)=P(E_5)=0.15 \quad P(E_3)=P(E_4)=0.25$$

Define events A, B, and C by

$$A = \{E_2,E_4,E_6\}, \; B = \{E_1,E_2,E_3\}, \; C = \{E_2,E_3,E_4,E_5\}.$$

Which events are independent? dependent?

$P(A) = .15 + .25 + .10 = .5$

$P(B) = .1 + .15 + .25 = .5$

$P(C) = .15 + .25 + .25 + .15 = .8$

A + B: $P(A) \cdot P(B) = P(A \cap B)$?

Dependent $.25 \neq P(E_2) = .15$

B + C: $P(B) \cdot P(C) = P(B \cap C)$

$.5 \quad .8 \quad P(E_2, E_3)$

Indep. $.4 \quad .15 + .25 = .4$

A + C:

$P(A) \cdot P(C) \quad P(A \cap C)$

$(.5)(.8) \quad P(E_2, E_4)$

$.4 = .15 + .25$

Independent

Some R commands

Entering a list:

```
>x=c(1,2,3)
```

or

```
>x<-c(1,2,3)

> x
[1] 1 2 3

>y=c(4,5,6)

> rbind(x,y)
  [,1] [,2] [,3]
x    1    2    3
y    4    5    6
```

```
> cbind(x,y)
     x y
[1,] 1 4
[2,] 2 5
[3,] 3 6

> a[2,2]
y
5
> a[1,3]
x
3

> sum(a)
[1] 21
```

```
> colSums(a)
[1] 5 7 9
> rowSums(a)
 x  y
 6 15
```

Let's put this in a table:

	Cash	Credit	Debit
<$20	.09	.03	.04
$20-$100	.05	.21	.18
>$100	.03	.23	.14

from 3.6 #1

What proportion of purchases are paid for using debit?

$.36$

Given that a purchase is for less than \$20, what is the probability that it is paid for by cash?

$$P(\text{cash} \mid < \$20) = \frac{P(\text{cash} \cap < \$20)}{P(< \$20)}$$

```
> sum(tbl[1,])
[1] 0.16
```

$$= \frac{.09}{.16} = 9/16$$

$.09 + .03 + .04 \rightarrow .16$

```
> tbl[1,1]/sum(tbl[1,])
  cash
0.5625
```

Are payment by cash and amount < \$20 independent events? NO

$$P(\text{cash}) \cdot P(< \$20) = P(\text{cash} \cap < \$20)$$

$$(.17)(.16) \neq .09$$