Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

Popper 05

1. If X is a discrete random variable, then $P(X \geq 2) =$

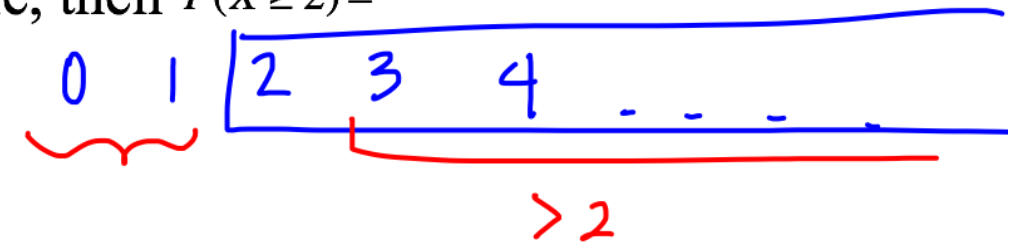
a. $1 - P(X \leq 2)$

b. $1 - P(X \leq 1)$

c. $P(X > 2)$

d. $1 - P(X < 3)$

e. none of these



If $X \sim \text{Binomial}(n, p)$, then

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad \text{where } x = 0, 1, 2, \ldots, n$$

$\binom{n}{x}$: nC_x

① prob of success is same for every trial
② fixed number of trials
③ each trial independent

R commands:

*
- $P(X = x) = dbinom(x, n, p)$
- $P(X \le x) = pbinom(x, n, p)$
- $P(X > x) = 1 - pbinom(x, n, p)$

$1 - P(X \le x)$

$$E[X] = np$$

$$\sigma^2 = np(1-p)$$

$\sigma = \sqrt{np(1-p)}$

Ex: Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. $n=15$ $p=.2$

Is X a binomial random variable? Yes — binom(,)

Determine the probability that exactly 3 fail the test.

$P(X=3) = {}_{15}C_3 (.2)^3(.8)^{12}$ = dbinom(3, 15, .2) = .2501

Determine the probability that at most 3 fail the test.

$P(X \le 3)$ = pbinom(3, 15, .2) = .6482

Determine the probability that at least 3 fail the test.

$P(X \ge 3) = 1 - P(X \le 2)$ = 1 - pbinom(2, 15, .2) = .6020

How many textbooks do we expect to fail the test? $E[X] = np = 15(.2) = 3$

What is the standard deviation of X? $\sigma = \sqrt{np(1-p)}$

$= \sqrt{15(.2)(.8)} = 1.549$

sqrt(15 * .2 * .8)

at most $\leq$

at least $\geq$

no more than $\leq$

less than $<$

no less than $\geq$

more than $>$

Ex: Each year a company selects a number of employees for a management training program. On average, 70% of those sent complete the program. Out of the seven people sent, what is the probability that

$p = .7$

$n = 7$

a. Exactly five complete the program?

$$P(X = 5) = dbinom(5, 7, .7) = .31765$$

b. Five or more complete the program?

$$P(X \geq 5) = 1 - P(X \leq 4) = 1 - pbinom(4, 7, .7) = .6471$$

same

$$1 - P(X < 5)$$

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c. more than 5 complete the program

$$P(X > 5) = 1 - P(X \leq 5) = 1 - pbinom(5, 7, .7) =$$

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> 1-pbinom(5,7,.7)
[1] 0.3294172
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Ex: Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that: $n=5$ $p=.8$

a. No one will contract the flu?

$P(X=0) = dbinom(0, 5, .8) = .00032$

b. All will contract the flu?

$P(X=5) = dbinom(5, 5, .8) = .3277$

c. Exactly two will get the flu?

$P(X=2) = dbinom(2, 5, .8) = .0512$

d. At least two will get the flu?

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - pbinom(1, 5, .8)$

$= .9933$

e. One or two get flu?

$P(X=1)$ & $P(X=2)$

2015 AP® STATISTICS FREE-RESPONSE QUESTIONS

3. A shopping mall has three automated teller machines (ATMs). Because the machines receive heavy use, they sometimes stop working and need to be repaired. Let the random variable X represent the number of ATMs that are working when the mall opens on a randomly selected day. The table shows the probability distribution of X.

Number of ATMs working when the mall opens	0	1	2	3
Probability	0.15	0.21	0.40	0.24

(a) What is the probability that at least one ATM is working when the mall opens?

$P(X \geq 1) = .21 + .4 + .24 = .85$ or $1 - P(X=0) = 1 - .15 = .85$

(b) What is the expected value of the number of ATMs that are working when the mall opens?

$E[X] = 0(.15) + 1(.21) + 2(.4) + 3(.24) = 1.73$

(c) What is the probability that all three ATMs are working when the mall opens, given that at least one ATM is working?

$P(X=3 \mid X \geq 1) = \dfrac{P(X=3 \text{ and } X \geq 1)}{P(X \geq 1)} = \dfrac{P(X=3)}{P(X \geq 1)} = \dfrac{.24}{.85} = .282$

(d) Given that at least one ATM is working when the mall opens, would the expected value of the number of ATMs that are working be less than, equal to, or greater than the expected value from part (b) ? Explain.

$(X=0 \mid X \geq 1)$ $(X=1 \mid X \geq 1)$ $(X=2 \mid X \geq 1)$ $(X=3 \mid X \geq 1)$

0 $\dfrac{.21}{.85}$ $\dfrac{.40}{.85}$ $\dfrac{.24}{.85}$

$E[X \mid X \geq 1] = 0.0 + 1(21/85) + 2(40/85) + 3(24/85) = 2.035$

The Hypergeometric Distribution

_hyper( )

Suppose that we have $m+n$ items and m have trait 1 while n have trait 2. We are interested in the probability that among k of the items, there are exactly x with trait 1. Is this a binomial distribution?

NO

m = trait want
n = trait dont want
K = # choosing from

Let X = the number of items among k with the desired trait.

Then X is said to have Hypergeometric Distribution with parameters m,n,k and

$$P(X=x)=\frac{\binom{m}{x}\binom{n}{k-x}}{\binom{m+n}{k}}, \quad x=0,1,2,\ldots,k$$

$$\frac{{}_mC_x \cdot {}_nC_{k-x}}{{}_{m+n}C_k}$$

R command is :

$$P(X=x)=dhyper(x,m,n,k)$$

$$P(X\le x)=phyper(x,m,n,k)$$

$$P(X>x)=1-phyper(x,m,n,k)$$

Example: A fish tank in a pet store has 24 fish in it. 7 are orange and 17 are white. Determine the probability that if we select 4 fish from the tank, at least 1 will be white.

$m = 17$

$n = 7$

$K = 4$

$P(X \geq 1) = 1 - P(X \leq 0) = 1 - \text{phyper}(0, 17, 7, 4)$

$1 - P(X = 0) = 1 - \text{dhyper}(0, 17, 7, 4)$

$= .9967$

more than 2 are orange? $m=7, n=17, K=4$

$P(X > 2) = 1 - P(X \leq 2) = 1 - \text{phyper}(2, 7, 17, 4)$

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> 1-phyper(2,7,17,4)
[1] 0.05928854
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Example: Suppose that 3 bank employees conspire to rob a particular bank branch where they work. The management is alerted to this fact, and gathers the 8 employees currently in the bank for interrogation. If the bank has 13 employees in total, determine the probability that the will find all 3 culprits among the 8 currently working? (What assumptions here are necessary?)

$m = 3 \quad n = 10 \quad K = 8$

$P(X = 3) = \text{dhyper}(3, 3, 10, 8) = .1958$

The **mean** of the a hypergeometric distribution is $E[Y] = kp = \frac{km}{m+n}$

and the variance is $Var(Y) = kp(1-p)\left(1 - \frac{k-1}{m+n-1}\right)$

Note: If $m+n$ is large compared to k (e.g. $m+n = 1000$, $k = 4$), this distribution may be *approximated* by a binomial distribution. What would the parameters be?

$n = K \qquad p = \frac{m}{m+n}$

Poppers 2-5 all A! Happy Friday!!