Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

Test 1:
In CASA
75 minutes
8 m/c at 7 pts each
4 f/r at 11 pts each
1 f/r has a bonus part worth 3 pts

Topics:
describing distributions
- skewness - what does it mean
- 5 num summary, IQR, outliers

Probability rules
Identify distributions
Solve probability problems
Expected value, variance, joint distributions, all that stuff...

Quick Review of Ch 4 – Discrete Distributions

Def: The ***probability mass function*** (pmf) of a discrete rv is defined for every number x_i by

$$f(x_i) = P(X = x_i)$$

$P(X<x) = P(X \le x-1)$

$P(X>x) = 1 - P(X \le x)$

Properties of f

$P(X=x)$

1. $f(x) \geq 0$ for all $x \in \mathbb{R}$
2. $\sum_x f(x) = 1$
3. $P(X \in A) = \sum_{x \in A} f(x)$, where $A \subseteq \mathbb{R}$ is a discrete set

1	2	3	4	5
		$X>4$		

Def: The ***cumulative distribution function*** (cdf) $F(x)$ of a discrete rv X with pmf $f(x)$ is defined for every number x by

$$F(x) = P(X \le x)$$

$F(3) = P(X \le 3)$

For any number x, $F(x)$ is the probability that the observed value of X will be at most x.

$E[X] = \sum x * p$

$V[X] = E[X^2] - E[X]^2$

Def: Let X be a discrete rv with set of possible values D and pmf p. The ***expected value*** or ***mean value*** of X, denoted E[X] or μ_X or just μ is $E\left[X\right]=\sum_{x\in D} x\cdot f\left(x\right)$

Properties of Expected Value and Variance

1. $E\left[c\right]=c$ for any constant $c\in\mathbb{R}$
2. $E\left[aX+bY\right]=aE\left[X\right]+bE\left[Y\right]$
3. $E\left[h\left(X\right)\right]=\sum_{x\in D} h\left(x\right)f\left(x\right)$ $\qquad E[X^2] = \sum x_i^2 \cdot p(x_i)$
4. $V\left(X\right)=E\left[\left(X-\mu\right)^2\right]$ or $V\left(X\right)=\underline{E\left[X^2\right]}-E\left[X\right]^2$
5. $V\left(aX+b\right)=a^2V\left(X\right)$

The Binomial Probability Distribution

A random variable X is a *Binomial* random variable if the following conditions are satisfied:

1. X represents the number of successes on n Bernoulli trials.
2. The probability of success for each trial is p.
3. The trials are mutually independent.

If X is a binomial random variable with probability p of success on each of n trials, we write $X \sim \text{Binomial}(n, p)$

If $X \sim \text{Binomial}(n, p)$, then

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad \text{where } x = 0, 1, 2, \ldots, n$$

R commands:

$$P(X = x) = dbinom(x, n, p)$$
$$P(X \le x) = pbinom(x, n, p)$$
$$P(X > x) = 1 - pbinom(x, n, p)$$

$$E[X] = np$$
$$\sigma^2 = np(1-p)$$

The Hypergeometric Distribution

Suppose that we have $m+n$ items and m have trait 1 while n have trait 2. We are interested in the probability that among k of the items, there are exactly x with trait 1. Is this a binomial distribution?

Let X = the number of items among k with the desired trait.

Then X is said to have Hypergeometric Distribution with parameters m,n,k and

$$P(X=x)=\frac{\binom{m}{x}\binom{n}{k-x}}{\binom{m+n}{k}}, \quad x=0,1,2,\ldots,k$$

R command is :

$$P(X=x)=dhyper(x,m,n,k)$$

$$P(X\le x)=phyper(x,m,n,k)$$

$$P(X>x)=1-phyper(x,m,n,k)$$

The **mean** of the a hypergeometric distribution is $E[Y]=kp=\dfrac{km}{m+n}$.

and the variance is $Var(Y)=kp(1-p)\left(1-\dfrac{k-1}{m+n-1}\right)$.

The Poisson Probability Distribution

A ***Poisson random variable***, X, represents the number of occurrences of a rare event during some fixed time period, where the expected number of occurrences is λ. (For example: The number of customers to enter a furniture store between 1:00pm and 2:00pm)

Conditions under which X is Poisson:

1. There is a fixed time period during which we are counting occurrences.
2. The expected number of arrivals during that time period is known to be λ.
3. The occurrences are independent of one another (the number of occurrences that have happened at any given point does not affect future occurrences).

If X has a Poisson distribution with parameter λ, then

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x=0,1,2,\ldots$$

where λ is a positive real number.

R commands:

$$P(X = x) = dpois(x, \lambda)$$

$$P(X \le x) = ppois(x, \lambda)$$

$$P(X > x) = 1 - ppois(x, \lambda)$$

For a Poisson random variable, X, with parameter λ

$E[X] = \lambda$, and $Var(X) = \sigma^2 = \lambda$

The Geometric Distribution

The **geometric distribution** is the distribution produced by the random variable X defined to count the number of trials needed to obtain the first success.

A random variable X is geometric if the following conditions are met:

1. Each observation falls into one of just two categories, "success" or "failure."
2. The probability of success is the same for each observation.
3. The observations are all independent.
4. The variable of interest is the number of trials required to obtain the first success.

The probability that the first success occurs on the n^{th} trial is

$$P(X = n) = (1 - p)^{n-1} p$$

where p is the probability of success.

The probability that it takes *more* than n trials to see the first success is

$P(X > n) = (1 - p)^n$

R commands: $P(X = n) =$ dgeom(n – 1, p)

$P(X \leq n) =$ pgeom(n – 1, p)

$P(X > n) =$ 1-pgeom(n – 1, p)

The mean, or expected number of trials to get a success in a geometric distribution is $E[X] = \mu = \dfrac{1}{p}$ and the variance is $\sigma^2 = \dfrac{1-p}{p^2}$.

Jointly Distributed Random Variables

We can display all the probabilities associated with jointly distributed random variables in a table (called a contingency table).

The probabilities in the middle of the table are called the *joint probabilities*.

The joint probability mass function is given by $f(x,y) = P(X = x, Y = y)$.

Properties of the joint probability mass function:

1. $0 \le f(x,y) \le 1$
2. $\Sigma\Sigma_{(x,y)\in\mathbb{R}^2} f(x,y) = 1$
3. $P((X,Y) \in A) = \Sigma\Sigma_{(x,y)\in A} f(x,y)$

$= P(X=0, Y=2)$

ind?

$P(X=0)\cdot P(Y=2) =$?

$y \backslash x$	0	1	2	marginal
1	.	.	.	$= P(Y=1)$
2				
3				
				1

The last row gives us the probabilities associated with X. These are called the *marginal probabilities*. The last column gives us the *marginal probability* for Y.

Calculation of marginal probabilities

$f_X(x) = P(X = x) = \Sigma_y f(x,y)$ and $\quad f_Y(x) = P(Y = y) = \Sigma_x f(x,y)$

Conditional Probabilities:

$$P(Y=y\,|\,X=x)=\frac{P(X=x,Y=y)}{P(X=x)}=\frac{f(x,y)}{f_X(x)}$$

$$P(X=x\,|\,Y=y)=\frac{P(X=x,Y=y)}{P(Y=y)}=\frac{p(x,y)}{p_Y(y)}$$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$P(A|B)\cdot P(B)=P(A\cap B)$$

Independence of Random Variables

We say that the random variables X and Y are *independent* if

$$f(x,y)=f_X(x)\,f_Y(y),\quad \text{for all } x,y$$

For joint random variables X and Y, the expected values are

$$\mu_X=E[X]=\sum_{(x,y)\in\mathbb{R}} x\,f(x,y)=\sum_x x\,f_X(x),$$

$$\mu_Y=E[Y]=\sum_{(x,y)\in\mathbb{R}} y\,f(x,y)=\sum_y y\,f_Y(y), \text{ and}$$

$$E[g(X,Y)]=\sum_{(x,y)\in\mathbb{R}} g(x,y)\,f(x,y)$$

Given two random variables X and Y, the *covariance of X and Y* is given by $\operatorname{cov}(X,Y)=E[XY]-E[X]E[Y]$ and the *correlation coefficient of X and Y* is given by $\rho=\dfrac{\operatorname{cov}(X,Y)}{\sigma_X\sigma_Y}$

$x\cdot y\cdot P(X=x,\ Y=y)$

Questions on Discrete RV?

5 blue
6 red
4 green
} 15

pick 7.

$$f(x, y) = \frac{x+y}{30}$$

$x = 0, 1, 2, 3$

$y = 1, 2, 3$

$y \backslash x$	

10 marbles

5 B   3R   2G

$$P(2B) = \frac{1}{2} \cdot \frac{4}{9}$$

Popper 09

1. Suppose X represents the number of tornadoes observed in Oklahoma during a one year period. If X has a Poisson distribution with $\lambda = 6$, find $P(3 \le X \le 8)$

   a. ppois(8,6)-ppois(3,6)
   b. ppois(8,6)-ppois(2,6)
   c. dpois(8,6)-dpois(3,6)
   d. dpois(8,6)-dpois(2,6)
   e. none of these

$1 \quad 2 \quad \boxed{3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8}$

2. Indicate which of the following distributions are binomial.

I. Draw marbles from a box containing equal numbers of blue and green marbles until you get a blue one.
II. Toss a balanced coin 10 times and count the number X of heads.
III. Roll a die until you get a six 3 times.

a. I only

b. II only

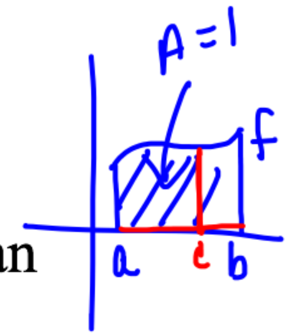
c. III only

d. I, II and III

e. II and III

f. None of these

Chapter 5 – Continuous Distributions not on exam 1



5.1 - Density Functions $f(x)$

A random variable X is *continuous* if it may assume any value in an interval, and assumes any *particular* value with probability 0.

For a continuous random variable, the probability that X is in any given interval is the integral of the probability density function over the interval.

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$\leq \quad <$ $\quad\quad \geq \quad >$

How does this differ from a discrete distribution's probability computation?

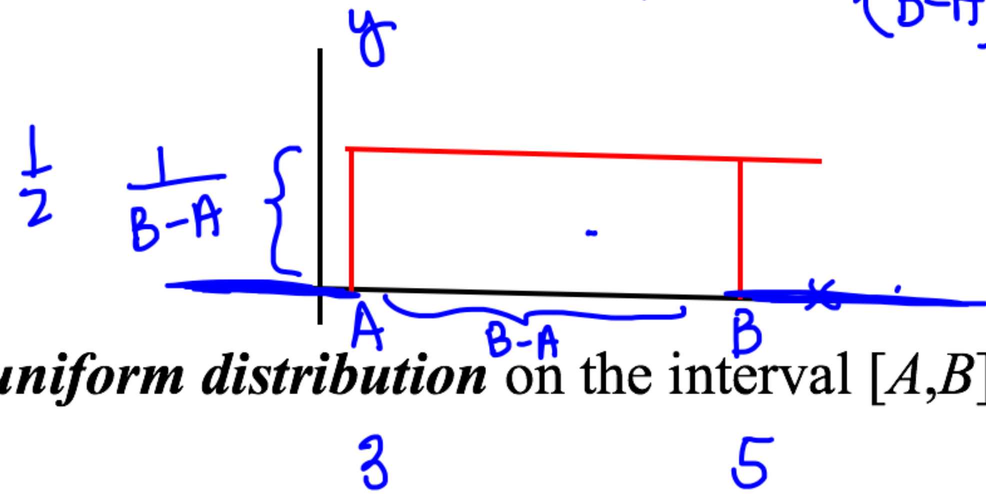
$P(X = c) = 0$

Also, since f is a probability density function, it must be that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Note: For any continuous probability distribution, $P(X = x) = 0$ for all x.
Why is this?

So this means that $P(a \le X \le b) = P(a < X < b)$.

$A = (B-A)\dfrac{1}{(B-A)} = 1$



The Uniform Distribution:

A continuous rv X is said to have ***uniform distribution*** on the interval $[A,B]$ if the pdf of X is given by

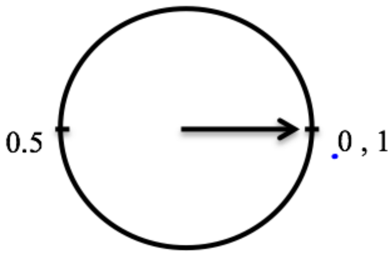
$$f(x; A, B) = \begin{cases} \dfrac{1}{B-A} & A \le x \le B \\ 0 & otherwise \end{cases}$$

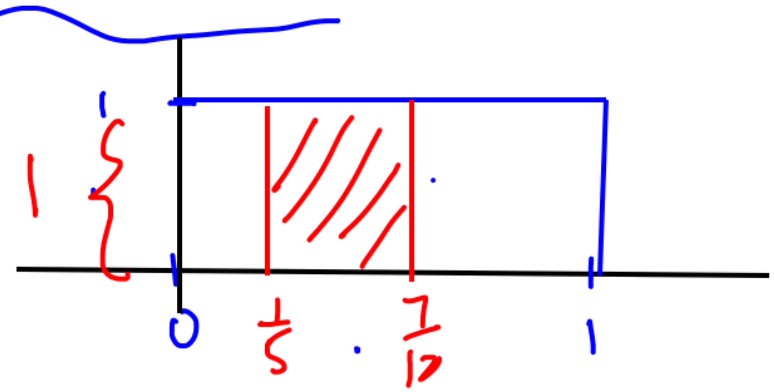
$$P(X \le x) = F(x) = \begin{cases} (x-A)\dfrac{1}{(B-A)} & A \le x \le B \\ 0 & x < A \\ 1 & x > B \end{cases}$$

cdf

$F(4)$

Example: Consider a spinner that, after a spin, will point to a number between zero and 1 with "uniform probability".





Define a probability density function for X, the result of the spin.

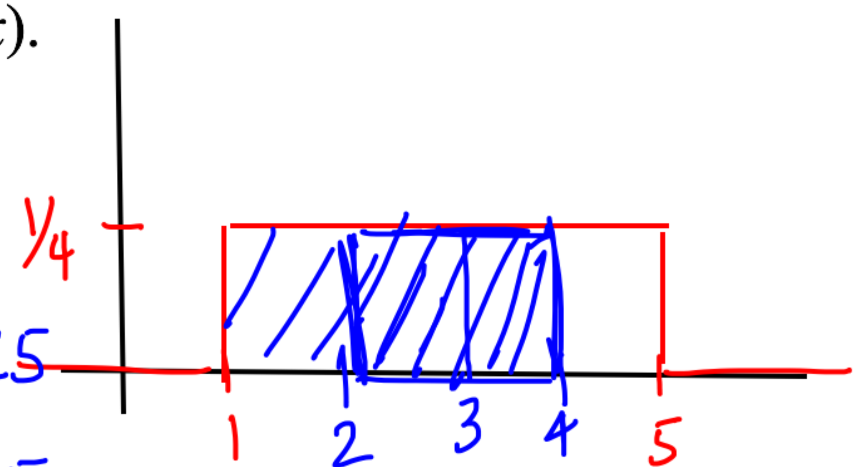
Determine $P\left(\frac{1}{5} \le X \le \frac{7}{12}\right)$.

$$\frac{7}{12} - \frac{1}{5} = \frac{35-12}{60} = \frac{23}{60}$$

$$= 1 \cdot \frac{23}{60} = \frac{23}{60}$$

Ex: A random variable is uniformly distributed over the interval [1,5]. Determine the pdf $f(x)$ and the cdf $F(x)$.

$$f(x) = \begin{cases} 1/4 & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$



$$F(x) = P(X \le x) = \begin{cases} (x-1)\frac{1}{4} & 1 \le x \le 5 \\ 0 & x < 1 \\ 1 & x > 5 \end{cases}$$

What is $P(X < 3)$? $= 2(1/4) = 1/2$

$P(X \le 4) = 3(1/4) = 3/4$

$P(X \le 0) = 0$

$P(X \le 1/2) = 0$

$P(X \le 5) = 1$

$P(X \le 6) = 1$

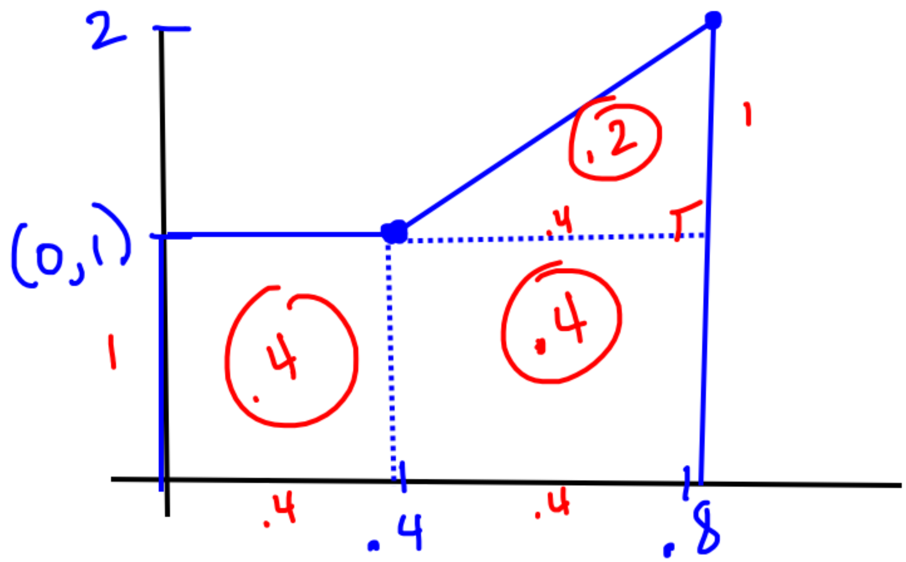
What is $P(2 \le X \le 4)$?

$2 \cdot 1/4 = 1/2$

Non-uniform example:

Think about a density curve for a continuous rv that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.

Sketch:



$.4 + .4 + .2 = 1$ ✓

What percent of observations fall below .4? $P(X < .4) = .4$

What percent of observations lie between .4 and .8? $P(.4 < X < .8) = .6$

What percent of observations are equal to .4? $0 = P(X = .4)$

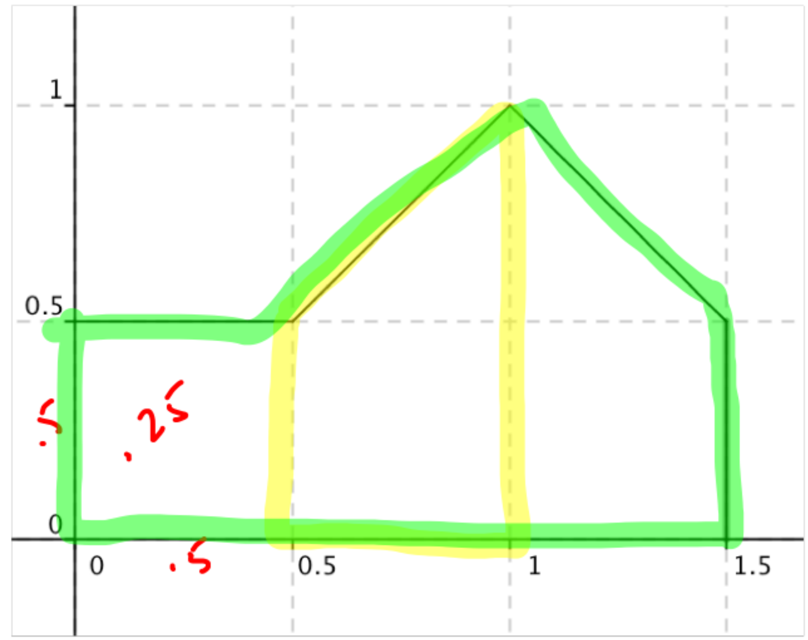
Popper 09

Use the following density curve to answer the questions:

3. $P(0.5 \le X \le 1) =$

4. $P(0 \le X \le 1.5) =$

5. $P(X = 1) =$



Use these choices for all 3:

a. 0.625
b. 0.25 1
c. 0.125
d. 0.375
e. 0
~~f. none of these~~

next time

Example: Suppose that the length X of the life (in years) of the field winding generator has a distribution that can be described by the pdf

$$f(x) = \frac{1.8x^{0.8}}{8^{1.8}} \exp\left[-\left(\frac{x}{8}\right)^{1.8}\right], \qquad 0 \le x < \infty$$

Determine the probability that a winding fails before the one year warranty expires on the machine.