# Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

## Popper 18

1. What effect does increasing the level of confidence have on the width of a confidence interval?  Does it make it longer, shorter, or stay the same?

   [✓ a. Longer]
   b. Shorter
   c. Stays the same

## Some General Concepts of Point Estimation

**Definition**

could be $\mu$ or $\sigma$ or $\beta$ or. $p$

A **point estimate** of a parameter $\theta$ is a single number that can be regarded as a sensible value for $\theta$.

some population value

sample value

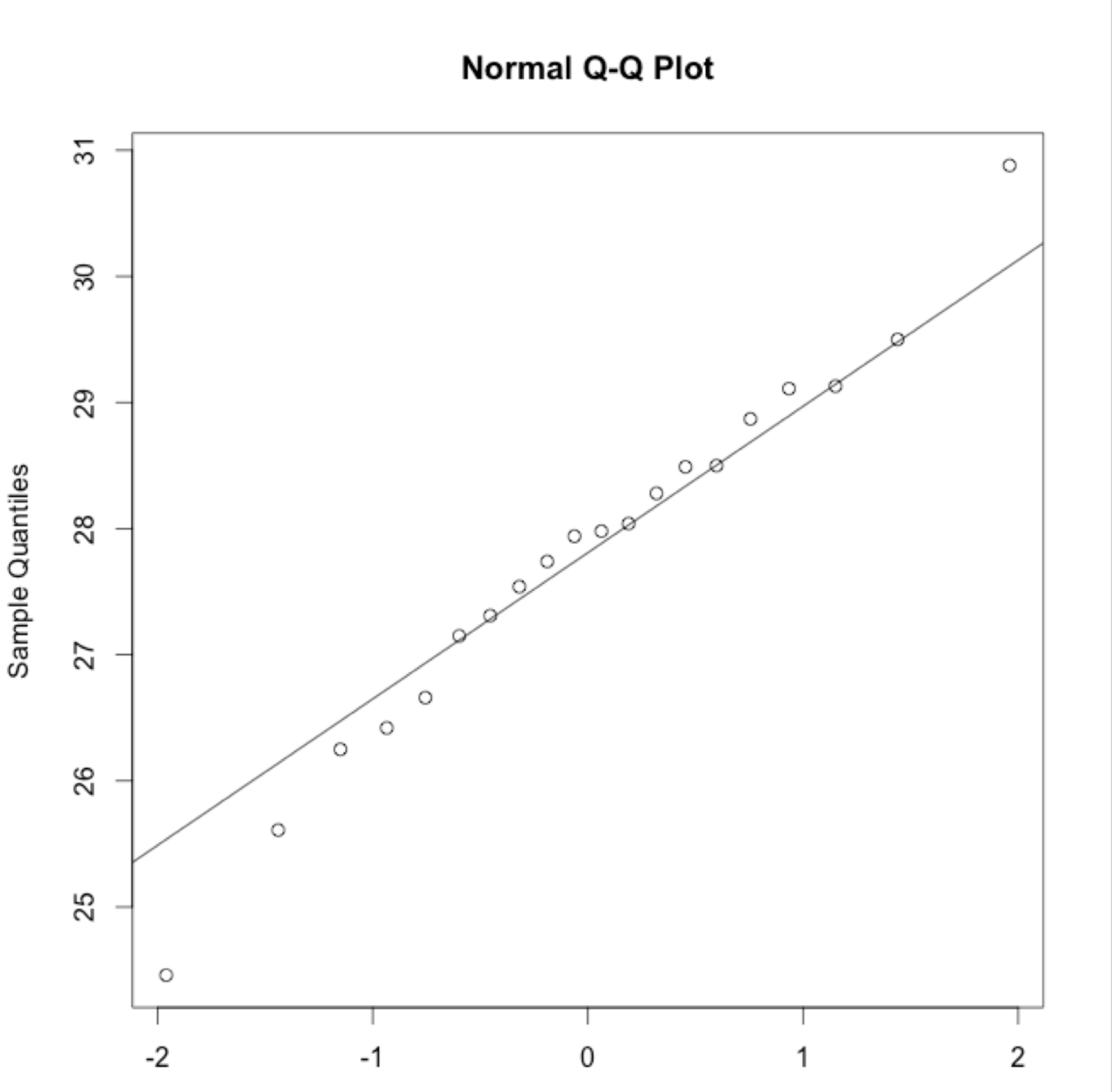
A point estimate is obtained by selecting a suitable statistic and computing its value from the given sample data. The selected statistic is called the **point estimator** of $\theta$.

**Example:** Consider the accompanying 20 observations on dielectric breakdown voltage for pieces of epoxy resin.

24.46 25.61 26.25 26.42 26.66 27.15 27.31 27.54 27.74
27.94 27.98 28.04 28.28 28.49 28.50 28.87 29.11 29.13
29.50 30.88

data = c(24.46, 25.61, ... )

```
> qqnorm(data)
> qqline(data)
```



Normal Q-Q Plot
Sample Quantiles
31
30
29
28
27
26
25
-2
-1
0
1
2

The pattern in the normal probability plot given there is quite straight, so we now assume that the distribution of breakdown voltage is normal with mean value $\mu$.

Because normal distributions are symmetric, $\mu$ is also the median lifetime of the distribution.

The given observations are then assumed to be the result of a random sample $X_1, X_2, \ldots, X_{20}$ from this normal distribution.
Consider the following estimators and resulting estimates for $\mu$ :

**a.** Estimator $= \overset{\text{mean}}{\bar{X}}$, estimate $= \bar{x} = \Sigma x_i / n = 555.86/20 = 27.793$

**b.** Estimator $= \overset{\text{median}}{\tilde{X}}$, estimate $= \tilde{x} = (27.94 + 27.98)/2 = 27.960$

**c.** Estimator $= [\min(X_i) + \max(X_i)]/2 =$ the average of the two extreme lifetimes,
estimate $= [\min(x_i) + \max(x_i)]/2 = (24.46 + 30.88)/2 = 27.670$

**d.** Estimator $= X_{\text{tr}(10)}$, the 10% trimmed mean (discard the smallest and largest 10% of the sample and then average),
estimator $= x_{\text{tr}(10)} = 27.838$

Each one of the estimators (a)–(d) uses a different measure of the center of the sample to estimate $\mu$. Which of the estimates is closest to the true value?

We cannot answer this without knowing the true value.

**Definition**

A point estimator  is said to be an **unbiased estimator** of $\theta$ if $E(\hat{\theta}) = \theta$ for every possible value of $\theta$. If $\hat{\theta}$ is not unbiased, the difference $E(\hat{\theta}) - \theta$ is called the **bias** of $\hat{\theta}$.

That is, $\hat{\theta}$ is unbiased if its probability (i.e., sampling) distribution is always "centered" at the true value of the parameter.

**Proposition**

When $X$ is a binomial rv with parameters $n$ and $p$, the sample proportion $\hat{p} = X/n$ is an unbiased estimator of $p$.

No matter what the true value of $p$ is, the distribution of the estimator $\hat{p}$ will be centered at the true value.

**Proposition**

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean $\mu$ and variance $\sigma^2$.

Then the estimator

$$\hat{\sigma}^2 = S^2 = \frac{\Sigma(X_i - \bar{X})^2}{n - 1}$$

is unbiased for estimating $\sigma^2$.

**Proposition**

If $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with mean $\mu$ , then $\bar{X}$ is an unbiased estimator of $\mu$. If in addition the distribution is continuous and symmetric, then $\tilde{X}$ and any trimmed mean are also unbiased estimators of $\mu$.

$n = 36$ $\bar{x}$

Ex: Suppose you administer a certain aptitude test to a random sample of 36 students in your school, and that the average score is 105. We want to determine the mean of the population of all students in the school. Assume a standard deviation of $\sigma = 15$ for the test.

qnorm$(1 - \alpha/2)$

$\alpha = 1 - CL$

$1 - \frac{(1-CL)}{2} = \frac{2-1+CL}{2} = \frac{1+CL}{2}$

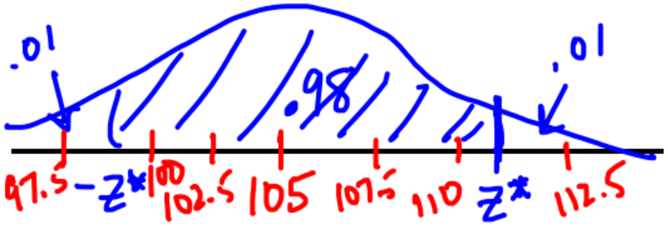
What is the z* value for a 98% confidence interval?

$z^* = \text{qnorm}(1.98/2)$

$2.326$

Sketch the distribution of sample means. Label the x-axis appropriately out to 3 standard deviations.

$S_{\bar{x}} = 15/\sqrt{36} = 2.5$



Determine the 98% confidence interval for the mean score $\mu$ for the whole school.

$\bar{x} \pm z^*\left(\frac{\sigma}{\sqrt{n}}\right) = \bar{x} \pm ME$

$105 \pm \underbrace{2.326\left(15/\sqrt{36}\right)}$

$(99.185, 110.815)$

What is the margin of error? → 5.815

Write a sentence that explains the significance of the level of confidence. (98%)

If we repeated this process 100 times, approximately 98 of the intervals found will contain the true population mean aptitude test score.

Write a sentence that explains the significance of the confidence interval.

We am 98% confident the true mean aptitude test score is between 99.185 and 110.815.

What sample size would be needed to have a margin of error at most 4 points?

$$ME = \underbrace{z^* \cdot \frac{\sigma}{\sqrt{n}}}$$

$$2.326\left(\frac{15}{\sqrt{n}}\right) \le 4$$

$$2.326\left(\frac{15}{4}\right) \le \sqrt{n}$$

$$\left[2.326\left(15/4\right)\right]^2 \le n$$

$$76.08 \le n \qquad \boxed{n = 77}$$

How would a confidence interval of this same data change if the confidence level were 95%, instead of 98%?

$\uparrow$

$z^* = 1.96$

Smaller interval

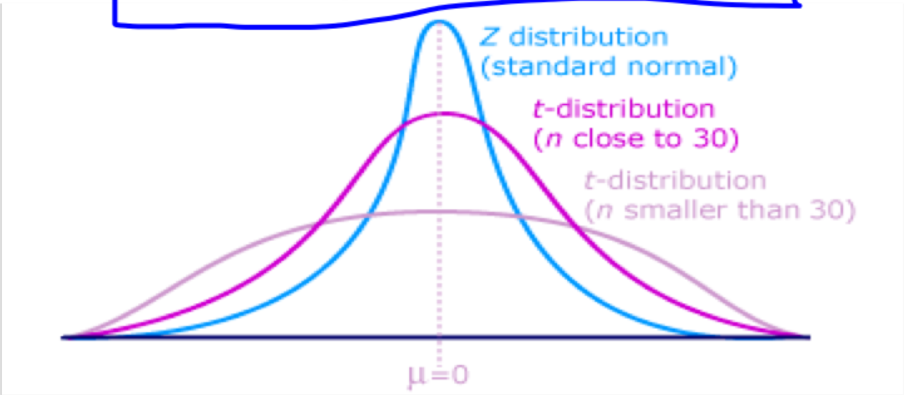
The assumption that we have made is that we know the standard deviation of the population. If we don't know the standard deviation of the distribution, we may use the *sample standard deviation,* $s^2$ as a point estimate for $\sigma^2$.

**The *t* distribution:**

Let $x_1, x_2, \ldots, x_n$ constitute a random sample from a normal population distribution. Then the probability distribution of the standardized variable

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

is the *t* distribution with $\boxed{(n-1) \text{ degrees of freedom.}}$



# Confidence Interval (with σ unknown)

When $\bar{x}$ is the sample mean from a population whose distribution is normal (or at least approximately normal), and σ is unknown, the $(1-\alpha)$ confidence interval for μ is the interval

$$\bar{x} \pm \underset{\hookrightarrow t^*}{t_{\alpha/2}}\frac{s}{\sqrt{n}}$$

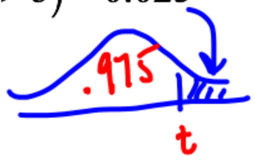
$$t^* = qt(1.CL/2\,,\, df)$$

Note: your book (and many others) allow the use of the normal distribution if n ≥ 30 when σ is unknown. We will just stick with using the t distribution when σ is unknown regardless of the sample size.

Example:

For a *t* distribution with $\underbrace{\text{8 degrees of freedom}}_{df = 8}$, find *c* so that $P(T > c) = 0.025$

(n = 9)



$qt(.975, 8)$

$= 2.306$

If *T* has 25 degrees of freedom, what would *c* be?

$qt(.975, 25) = 2.0595$

$\sigma =$ population st. dev. $\quad s =$ sample st. dev.

Example: The mean μ of the times it takes a lab technician to perform a certain task is of interest. If the lab technician was observed on $n = 16$ different occasions and the mean and standard deviation of these times were $\overset{\bar{x}}{4.3}$ minutes and $\overset{s}{0.6}$ minutes respectively. Give a 95% confidence interval for her mean completion time μ.

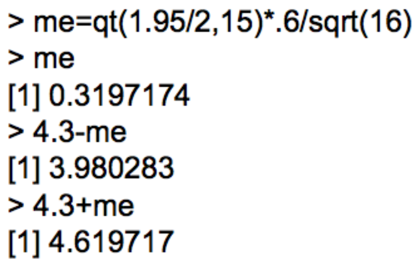
$$t^* = qt(1.95/2, 15)$$

$$t^* = 2.13145$$

$$\bar{x} \pm t^*\left(\frac{s}{\sqrt{n}}\right)$$

$$4.3 \pm 2.13145\left(\frac{.6}{\sqrt{16}}\right)$$

$$(3.98, 4.62)$$



```
> me=qt(1.95/2,15)*.6/sqrt(16)
> me
[1] 0.3197174
> 4.3-me
[1] 3.980283
> 4.3+me
[1] 4.619717
```

Example: The mean breaking strength $\bar{x}$ of a sample of $\underbrace{n = 112}$ steel beams is 42,196 pounds per square inch, and the sample standard deviation is $s = 614$. Provide a 90% confidence interval for the mean breaking strength μ. Provide a 95% confidence interval for μ.

$\bar{X} = 42196$ $s = 614$ 90% $\Rightarrow t^* = qt(1.9/2, 111) = 1.66$

90% C.I. $42196 \pm 1.66\left(\frac{614}{\sqrt{112}}\right) \rightarrow (42099.77, 42292.23)$

95% CI $(42081.03, 42310.97)$

## Popper

qt(1.99/2, 8)
= 3.355
s

2. The weight of 9 men have mean 175 lbs. and a standard deviation 15 lbs. What is the margin of error of the mean for a 99% confidence interval?

a. 12.879 b. 19.445 c. 5 [✓ d.] 16.777

3. A sample of $n = 16$ completion times for a particular task for a lab technician led to a sample mean of 4.3 minutes and a sample standard deviation of 0.6 minutes. Determine a 95% confidence interval for the variance of her completion time.

- a. [.443, .928]
- b. [.196, .862]
- c. [6.26, 27.49]
- d. [.134, .996]
- e. none of these

3. A

## Hypothesis Testing

A statistical hypothesis is simply a statement about one or more distributions or random variables.

*Statistical testing of hypotheses* is an area of statistics that deals with procedures for confirming and refuting hypotheses about distribution of random variables.

$H_0: \mu = 84$

$H_a: \mu < 84$

Def: The ***null hypothesis***, denoted by $H_0$, is the claim that is initially assumed to be true. The ***alternative hypothesis***, denoted by $H_a$ or $H_1$ is the assertion that is contradictory to $H_0$.

The null hypothesis will be rejected in favor of the alternative hypothesis only if sample evidence suggests that $H_0$ is false.

If the sample evidence does not *strongly contradict* $H_0$, we will continue to believe in the plausibility of the null hypothesis.

Hypothesis Testing has 2 possible conclusions:

1. Reject $H_0$
2. Fail to reject $H_0$

The Test Procedure:

1. A ***test statistic*** which is calculated from the sample data, and
2. A ***rejection region***, which is based on the distribution and the desired confidence level.

We will reject $H_0$ if the test statistic lies in the rejection region.

## Possible Errors and Operating Characteristic Curves

Type I error: rejecting $H_0$ when $H_0$ is true.
Type II error: failing to reject $H_0$ when $H_a$ is true.

In other words,
A **type I error** occurs when you reject a true null hypothesis.
A **type II error** occurs when you fail to reject a false null hypothesis.

We need to try our best to have procedures such that either type of error is unlikely to occur.

Let's look at this:

| | | **Truth** | |
|---|---|---|---|
| | | Person is innocent | Person is guilty |
| **Decision** of Jury | Guilty | *Error* | *Correct Decision* |
| | Not Guilty | *Correct Decision* | *Error* |

In general:

| | | **Truth**<br>(for population studied) | |
|---|---|---|---|
| | | $H_0$ True | $H_0$ False |
| **Decision**<br>(based on sample) | Reject $H_0$ | *Type I Error* | *Correct Decision (Power)* |
| | Fail to reject $H_0$ | *Correct Decision* | *Type II Error* |

By determining $\alpha$ (our level of significance) we can try to control for the type 1 error.

$$\alpha = P(\text{type I error}) = P(H_0 \text{ is rejected when it is true}) = P(H_0 \text{ is rejected} \mid H_0 \text{ true})$$
$$\beta = P(\text{type II error}) = P(H_0 \text{ is not rejected when it is false}) = P(H_0 \text{ is NOT rejected} \mid H_0 \text{ false})$$

The Power of a test against an alternative hypothesis is the probability that a fixed level $\alpha$ significance test will reject the null hypothesis when that particular alternative is true.

→ Power = 1 - P(Type II error)

**$H_0$: I will be better off if I take no action.**
**$H_a$: I will be better off if I take action.**

**Type I Error** would correspond to taking action when you would have been better off not doing so.
**Type II Error** would correspond to taking no action when you would have been better off taking action.
**Power** would correspond to taking action when you would have been better off taking action.

4-5 A