Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

when not stated, $\alpha = .05$

Hypothesis Testing about a Population Mean

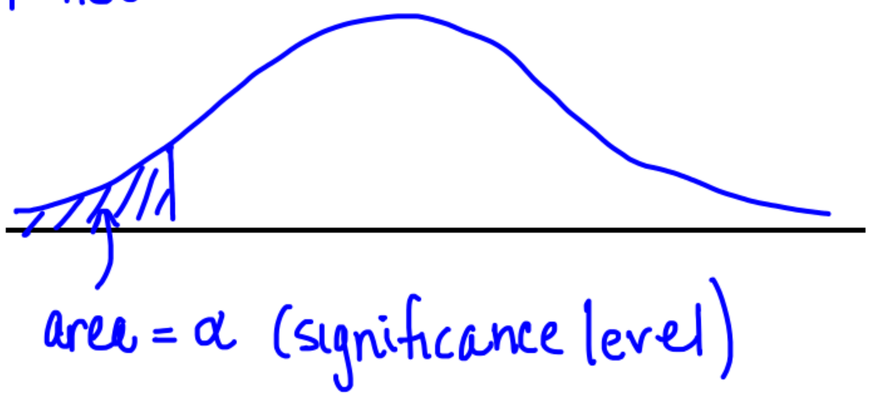
For a test with fixed sample size, n, and given significance level, α, we draw a *rejection region* as a guideline for when the null hypothesis may be rejected.

Example: Sketch the rejection region for the hypotheses

claimed pop. mean

$$H_0 : \mu = \mu_0$$

$$H_a : \mu < \mu_0$$



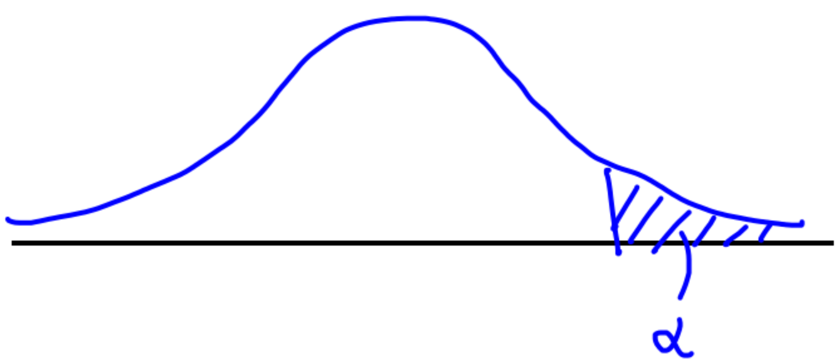
ex: $H_0: \mu = 10$

$H_a: \mu < 10$

Example: Sketch the rejection region for the hypotheses

$$H_0 : \mu = \mu_0$$

$$H_a : \mu > \mu_0$$

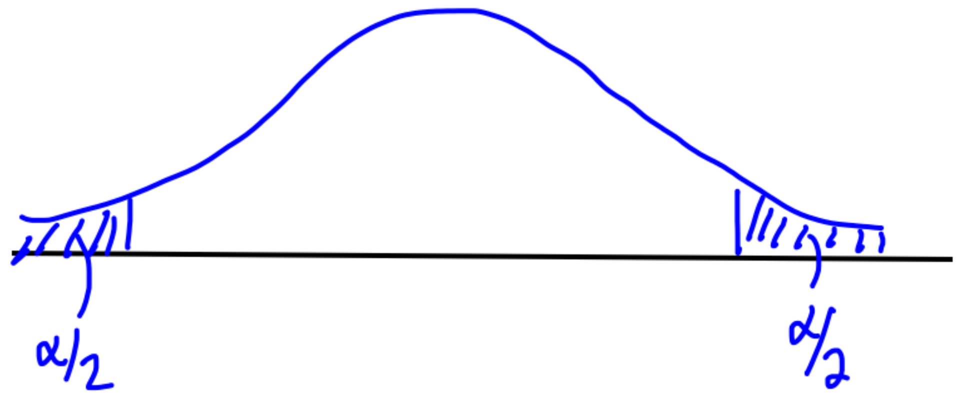


Example: Sketch the rejection region for the hypotheses

$$H_0 : \mu = \mu_0$$

$$H_a : \mu \neq \mu_0$$

different



Test Statistics for μ:

1. Normal Population, σ known: $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

pop. st. dev.

2. Normal Population, σ unknown: $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$

Def: The ***P* – *value*** is the probability, calculated assuming that the null hypothesis is true, of obtaining a value of the test statistic at least as contradictory to H_0 as the value calculated from the available sample.

P (our sample yelded given results | H_0 is true)

Proposition: The *P* – value is the smallest significance level α at which the null hypothesis can be rejected. Thus, the *P* – value is sometimes referred to as the ***observed significance level*** (OSL) for the data.

Example: The tar content of a certain type of cigarette has been averaging 11.5 milligrams per cigarette with a standard deviation of 0.6 milligram. A researcher has discovered a new filter that he claims will reduce the mean tar content from 11.5. That is, he claims that if μ is the mean tar content of cigarettes with the new filter, $\mu < 11.5$.
We consider $n = 50$ randomly selected cigarettes having the new filter and find that the average tar content is $\bar{x} = 11.4$. Is this enough evidence to reject $H_0: \mu = 11.5$ at the $\alpha = 0.10$ significance level?

$H_0: \mu = 11.5$

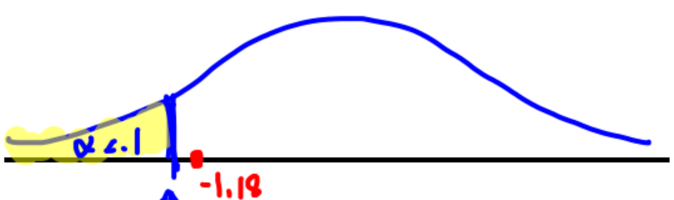
$H_a: \mu < 11.5$

$\bar{x} = 11.4$

σ or s ?

use z

$$z = \frac{11.4 - 11.5}{.6/\sqrt{50}} = -1.18$$



qnorm(.1) = -1.28

with 90% certainty

Based on $\boxed{\alpha = .1}$, we will fail to reject the claim (H_0) that the mean tar content is 11.5 mg per cigarette.

pvalue: $P(z < -1.18) = \text{pnorm}(-1.18) = 11.9\%$

.119

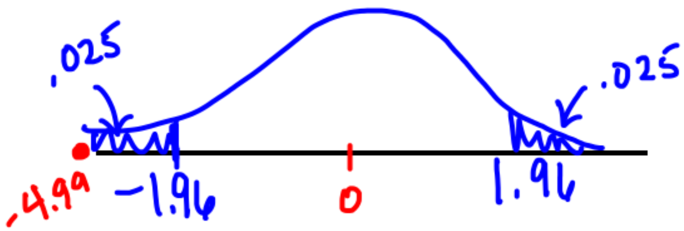
Example: A laboratory is asked to evaluate the claim that the concentration of the active ingredient in a specimen is 0.86 with a standard deviation of 0.0068. The mean of three repeated analyses of the specimen is $\boxed{\bar{x} = 0.8404.}$ Do these analyses indicate that the concentration of the active ingredient is different than the original claim? ($\alpha = .05$)

μ n σ $\neq$

$H_0: \mu = .86$

$H_a: \mu \neq .86$

$$z = \frac{(.8404 - .86)}{(.0068/\sqrt{3})} = -4.99$$



pvalue: $p(z < -4.99) \cdot 2 \approx .000 < .05$

1% or 99.9% confident

Based on a 5% sign. level we can reject the claim that the mean concentration of active ingredient of specimen is .86 in favor of saying it is different.

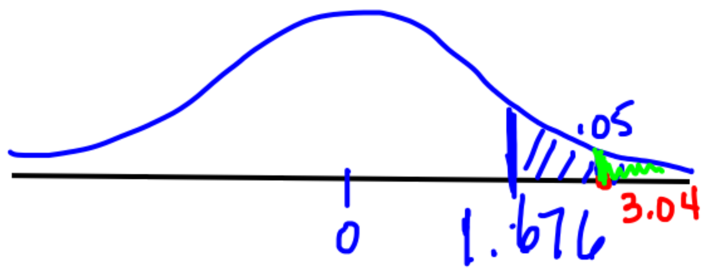
Reject H_0 when pvalue $< \alpha$
(and test statistic in shaded rejection region)

Urban storm water can be contaminated by many sources, including discarded batteries which release metals when ruptured. A sample of 51 = n Panasonic AAA batteries gave a sample mean zinc mass of 2.06 g ($\bar{x}$) and a sample standard deviation of 0.141 g.
Does this data provide compelling evidence for concluding that the population mean zinc mass exceeds 2.0 g? $\alpha = .05$

$H_0: \mu = 2.0$

$H_a: \mu > 2.0$

z or t?



qt(.95, 51-1)

Reject H_0

$$t = \frac{2.06 - \boxed{2.0}}{.141/\sqrt{51}} = 3.04$$

pvalue: $p(t > 3.04) = 1 - pt(3.04, 50)$

$= .0019 < \alpha$

Based on 99% confidence we can reject H_0 which states the zinc mass is 2.0g in favor of saying it is greater.

Ex: The target thickness for silicon wafers used in a certain type of integrated circuit claimed to be 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean thickness of 246.18 μm and a sample standard deviation of 3.60 μm. Does this data suggest (at 99% confidence level) that true average wafer thickness is something other than the target value? $\alpha = .01$

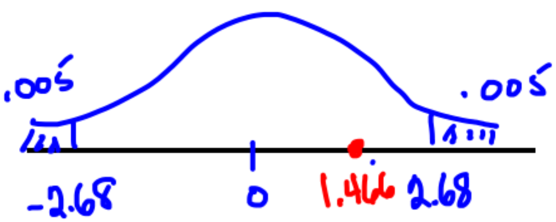
$\neq$

$H_0: \mu = 245$

$H_a: \mu \neq 245$

$n = 50$ qt(.005,49)

$$t = \frac{246.18 - 245}{3.6/\sqrt{50}} = 1.466$$



Fail to reject H_0

pvalue: $2P(t > 1.466) =$

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> 2*(1-pt(1.466,49))
[1] 0.1490366
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Fri.

The belief is that the mean number of hours per week of part-time work of high school seniors in a city is 10.6 hours. Data from a simple random sample of 50 seniors indicated that their mean number of part-time work was 12.5 with a standard deviation of 1.3. Test whether these data cast doubt on the current belief ($\alpha = .05$)

Suppose we have a sample of $n = 49$ whose population standard deviation is $\sigma = 20$ and we wish to test the following at $\alpha = 0.07$:

$$H_0 : \mu = 55$$
$$H_a : \mu < 55$$

This is a z-test. For what values of z will we reject the null hypothesis?

For what values of $\bar{x}$ will we reject the null hypothesis?

What is the value of the probability of the type I error?

Now suppose that the population mean has been determined to be $\mu = 47$.
What is P(Type II error)?

$$P(H_0 \text{ is NOT rejected} \mid \mu = 47)$$

What is the probability of the Power?

Popper 19

At the bakery where you work, loaves of bread are supposed to weigh 1 pound, with standard deviation σ =0.13 pounds. You believe that new personnel are producing loaves that are lighter than 1 pound. As supervisor of Quality Control, you want to test your hypothesis at the 95% confidence level. You weigh 20 loaves and obtain a mean weight of 0.95 pounds.

1. This is a ___ test
   a. z b. t
2. The probability of a type 1 error is
   a. .95 b. .05 c. .025 d. none of these
3. The null and alternate hypothesis would be
   a. $H_0: \mu = .95$, $H_a: \mu < .95$ b. $H_0: \mu = 1$, $H_a: \mu < 1$ c. $H_0: \mu = 1$, $H_a: \mu < .95$

4. Rejecting a true null hypothesis is classified as a _______.

   a. Type I error   b. Type II error   c. Power

5. Failing to reject a false null hypothesis is classified as a _______________.

   a. Type I error   b. Type II error   c. Power

6. The level of significance α for a hypothesis test is equal to the probability of a _____.

   a. Type I error   b. Type II error   c. Power