Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

Hypothesis Testing about a Population Mean

The belief is that the mean number of hours per week of part-time work of high school seniors in a city is 10.6 hours. Data from a simple random sample of 50 seniors indicated that their mean number of part-time work was 12.5 with a standard deviation of 1.3. Test whether these data cast doubt on the current belief $(\alpha = .05)$

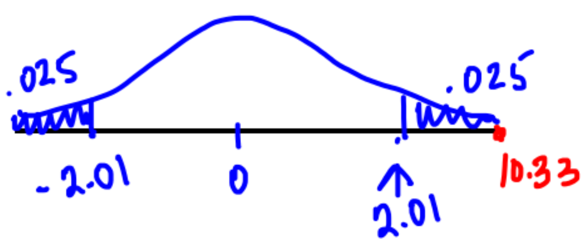
$n = 50,\ \bar{x} = 12.5,\ s = 1.3$

$H_0: \mu = 10.6$

$H_a: \mu \boxed{\neq} 10.6$

$t^* = qt(.025 \text{ or } .975,\ \underset{n-1}{49}) =$

$\left(\alpha/2 \text{ or } 1-\alpha/2 \text{ since } H_a \neq\right)$

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> qt(.975,49)
[1] 2.009575
```

95% $1-\alpha$

$$t = \frac{12.5 - 10.6}{1.3/\sqrt{50}} = 10.33$$

Based on 99% confidence, I will reject the claim that the mean hours worked for these high school seniors is 10.6hr in favor of saying its different.

pvalue $2 \cdot p(t \geq 10.33) = 2*(1 - pt(10.33, 49)) \approx .000 < \alpha$

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> 2*(1-pt(10.33,49))
[1] 6.77236e-14
```

Reject H_0

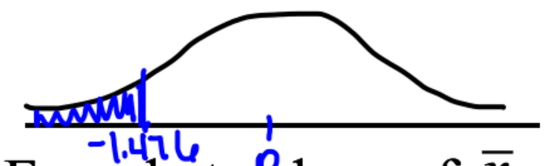
Suppose we have a sample of $n = 49$ whose population standard deviation is $\sigma = 20$ and we wish to test the following at $\alpha = 0.07$:

qnorm(.07)

$$H_0 : \mu = 55$$
$$H_a : \mu < 55$$

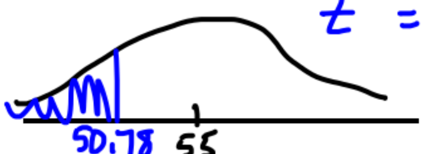
This is a z-test. For what values of z will we reject the null hypothesis?

reject if $z < -1.476$



For what values of $\bar{x}$ will we reject the null hypothesis?

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad -1.476 = \frac{\bar{x} - 55}{20/\sqrt{49}} \Rightarrow \bar{x} = 50.78$$

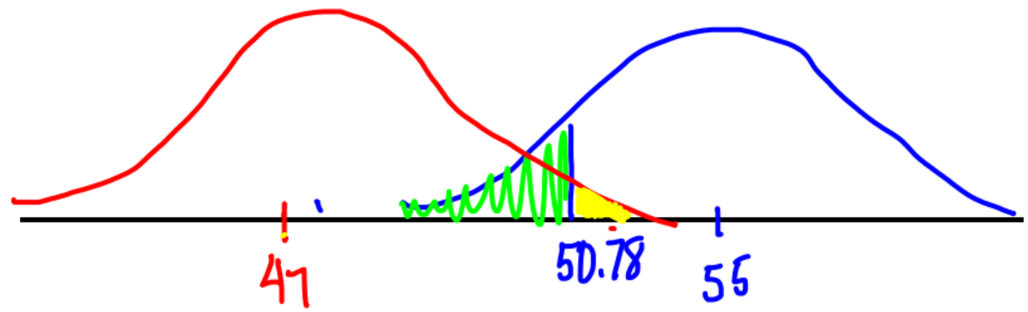


What is the value of the probability of the type I error? $= \alpha$

P(type I error) = .07

Now suppose that the population mean has been determined to be $\mu = 47$.
What is P(Type II error)?

$\underbrace{P(H_0 \text{ is NOT rejected}}\,|\,\mu = 47)$



$P(\bar{x} > 50.78 \mid \mu = 47)$

$P\left(z > \frac{50.78 - 47}{20/\sqrt{49}}\right) = P(z > 1.323) = 1 - \text{pnorm}(1.323) = .0929$

$\underline{\underline{9.3\%}}$

What is the probability of the Power?

$P(\text{power}) = 1 - P(\text{type II error})$

$1 - .0929$

$= .9071 = 90.7\%$

Popper 20

1. A t – test is used instead of a z – test when
   a. The population mean is unknown
   b. The population size is unknown
   c. The population standard deviation is unknown.
   d. None of these

2. Suppose a test of the hypothesis in a question yields a p – value of 0.02. Based on a 5% significance level, you would
   a. Reject the null hypothesis
   b. Fail to reject the null hypothesis
   c. Accept the null hypothesis
   d. None of these

pvalue = .02

α = .05

Confidence Intervals about a Population Proportion

single

Suppose we have a population in which each member either has a trait or does not have the trait. We may obtain a confidence interval for the *proportion* of the population with the trait, p.

$\hat{p}$ = sample proportion
= x/n

If a sample of size n is taken from our population, and y of the sample have the trait, then an *approximate* $(1-\alpha)100\%$ confidence interval for the proportion p of the population with the trait is

$$\hat{p} \pm \underset{z^*}{z_{\alpha/2}} \underbrace{\sqrt{\frac{\hat{p}\hat{q}}{n}}}_{\text{st. error}}$$

where $\hat{p} = \dfrac{y}{n}$ and $\hat{q} = 1 - \hat{p}$

Choosing the sample size:

$$z^* \sqrt{\frac{p^*(1-p^*)}{n}} \le m$$

Use p^*=.5 if you don't have a better guess.

n $\qquad\qquad$ $\hat{p} = \frac{104}{200} = .52$

Example: Suppose we interview 200 voters and 104 say they plan to vote for a certain candidate. Determine a 90% confidence interval for the proportion of the population that plan to vote for the candidate.

90% $\Rightarrow \alpha = .1$

$z^* =$ qnorm(.95)

```
> me=qnorm(.95)*sqrt(.52*.48/200)
> me
[1] 0.05810782
> .52-me
[1] 0.4618922
> .52+me
[1] 0.5781078
```

$$.52 \pm 1.645 \cdot \sqrt{\frac{(.52)(.48)}{200}}$$

$(.462, .578)$

$n=100 \quad \hat{p}=.54$

Example: An experimenter flips a coin 100 times and gets 54 heads. Find an approximate 90% confidence interval for the probability of flipping a head with this coin.

$z^*=1.645$

$$.54 \pm 1.645\sqrt{\frac{.54(.46)}{100}}$$

```
> me=1.645*sqrt(.54*.46/100)
> .54-me
[1] 0.4580136
> .54+me
[1] 0.6219864
```

$(.458, .622)$

Suppose that prior to conducting the coin-flipping experiment, we suspect that the coin is fair. How many times would we have to flip the coin in order to obtain a 90% confidence interval of width of at most 0.1 for the probability of flipping a head?

$ME = .05$

$\underbrace{ME \quad -\hat{p}+ \quad ME}_{\text{width}}$

$$1.645\sqrt{\frac{.5\cdot .5}{n}} \le .05$$

$$\frac{.5}{\sqrt{n}} \le \frac{.05}{1.645}$$

$$\frac{.5(1.645)}{.05} \le \sqrt{n}$$

$$270.6 \le n$$

$$n = 271$$

Monday

Hypothesis Testing about a Population Proportion

Test Statistic for p:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

Example: Let p be the average fraction of "grade A" items produced by a certain company. The company claims that their factory produces 62% grade A items. To test the claim that $p = 0.62$ against the alternative $p \neq 0.62$, a sample of 250 items is taken, and 172 grade A items are found. What do we conclude?

Pizza Hut, after test-marketing a new product called the Bigfoot Pizza, concluded that introduction of the Bigfoot nationwide would increase it's sales by more than 14% (*USA Today*, April 2, 1993). This conclusion was based on recording sales information for a random sample of Pizza Hut restaurants selected for the marketing trial. With π denoting the percentage increase in sales for all Pizza Hut restaurants, consider using the sample data to decide between

$$H_0: \pi = 14\% \qquad \text{vs.} \quad H_a: \pi > 14\%$$

a) If Pizza Hut is incorrect in its conclusion is the company making a type I or a type II error?

b) What are possible consequences of this error?

3, 4, 5, 6 = C