

Math 3339

Section 27204

MWF 10-11:00am AAAud 2

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Office Hours:

M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

3%

A manufacturer of circuits had observed that, on average, $p = 0.03$ of its circuits failed. One of the engineers suggests changes in the design to try to improve this percentage.

It is decided that $n = 100$ circuits would be made using her method. The company will adopt her method if only zero or one of the circuits failed.

Identify the null and alternative hypotheses at play here. What are the possible Type I and Type II errors the company might make with this strategy?

$$H_0: p = .03$$

$$H_a: p < .03$$

Type I error: adopt new method when there is still $p = .03$ failure rate
Type II error: failed to adopt new method when it would have worked better ($p < .03$)

If the actual value for the new method is $p = 0.01$, what is the probability that the company will make a Type II error?

Fail to adopt if 2 or more circuits out of $n = 100$ fail

$$\begin{aligned} P(X \geq 2, p = .01) &= 1 - P(X \leq 1) && 1 - \text{pnorm}(1, 100, .01) \\ &= 1 - (P(X = 0, p = .01) + P(X = 1, p = .01)) \\ &= 1 - ({}^{100}C_0 (.01)^0 (.99)^{100} + {}^{100}C_1 (.01)^1 (.99)^{99}) \\ &= .264 \end{aligned}$$

Mixed Examples:



conf interval $g - \left(\frac{1+CL}{2} \right)$

Patients with chronic kidney failure may be treated by dialysis, using a machine that removes toxic wasters from the blood, a function normally performed by the kidneys. Kidney failure and dialysis can cause other changes, such as retention of phosphorous, that must be corrected by changes in diet. A study of the nutrition of dialysis patients measured the level of phosphorous in the blood of several patients on six occasions.

Here are the data for one patient:

5.6 5.3 4.6 4.8 5.7 6.4

The measurements are separated in time and can be considered a SRS of the patient's blood phosphorous level. $\rightarrow Z$

If this level varies normally with $\sigma = 0.9$, give a 90% confidence interval for the mean blood phosphorous level and **interpret your results**.

```
> kidney=c(5.6,5.3,4.6,4.8,5.7,6.4)
```

```
> mean(kidney)
```

```
[1] 5.4
```

```
> sd(kidney)
```

```
[1] 0.6542171
```

```
> zstar=qnorm(1.9/2)
```

```
> zstar
```

```
[1] 1.644854
```

```
> me=zstar*.9/sqrt(6)
```

```
> me
```

```
[1] 0.6043578
```

```
> 5.4-me
```

```
[1] 4.795642
```

```
> 5.4+me
```

```
[1] 6.004358
```

(4.796, 6.004)

I am 90% confident the true mean blood phosphorous level is between 4.796 and 6.004

← only if need if not given

$$CL + d/2$$

$$1 - d = CL$$

$$1 - CL = d$$

$$CL + \frac{(1 - CL)}{2} = \frac{2CL}{2} + \frac{1 - CL}{2}$$

$$= \frac{1 + CL}{2}$$

assume σ not given
s

```
> t.test(kidney, conf.level=.90)
```

One Sample t-test

data: kidney

t = 20.218, df = 5, p-value =
5.473e-06

alternative hypothesis: true mean is
not equal to 0

90 percent confidence interval:

(4.861815, 5.938185)

sample estimates:

mean of x

5.4

```
> me=qt(1.9/2,5)*sd(kidney)/sqrt(6)
```

```
> me
```

```
[1] 0.5381852
```

```
> 5.4-me
```

```
[1] 4.861815 ←
```

```
> 5.4+me
```

```
[1] 5.938185 ←
```

$$\alpha = .05$$

Is there strong evidence that the patient has a mean phosphorous level that exceeds 4.8? (Show all necessary steps in determining your answer and state your conclusions in complete sentences.)

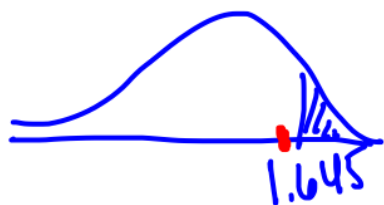
$$H_0: \mu = 4.8$$

$$H_a: \mu > 4.8$$

$$\bar{x} = 5.4$$

$$\sigma = .9$$

One sample mean z test



$$z = \frac{5.4 - 4.8}{.9 / \sqrt{6}} = 1.63$$

```
> (5.4-4.8)/(.9/sqrt(6))
[1] 1.632993
```

$$p\text{ value: } p(z > 1.63) = .051 > \alpha$$

```
> 1-pnorm(1.633)
[1] 0.0512345
```

Based on $\alpha = .05$, we will fail to reject the claim that the mean blood phosphorus level is 4.8.

If we did a t-test :

```
> (5.4-4.8)/(sd(kidney)/sqrt(6))  
[1] 2.246493 ← test stat  
> 1-pt(2.246,5)  
[1] 0.03732525 ← pvalue
```

$$\mu = 98 \quad \sigma = 10$$

pop. It is believed that the average amount of money spent per U.S. household per week on food is about \$98, with standard deviation \$10. A random sample of 100 households in a certain affluent community yields a mean weekly food budget of \$100. \bar{x} We want to test the hypothesis that the mean weekly food budget for all households in this community is higher than the national average.

Are the results significant at the 5% level? Explain.

$$H_0: \mu = 98$$

$$H_a: \mu > 98$$



$$Z = \frac{100 - 98}{10 / \sqrt{100}} = 2$$

$$p\text{value} = 1 - p\text{norm}(2) = .023 < \alpha$$

α

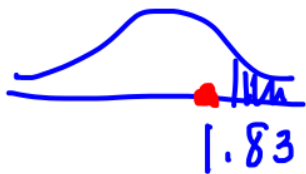
Based on 5% sign level we can reject the null hyp. and conclude the avg amount spent on groceries in this comm. is higher than the average of \$98.

A company claims that the mean deflection of its 10-foot steel beams is 0.012 inch. A construction contractor who purchases large quantities of steel beams suspects that the manufacturer misleads its customers and that the true deflection is, in fact, larger than the one claimed. To see whether this suspicion is justified, the contractor selects $n=10$ beams at random from his inventory, determines their deflection, and conducts a test to determine if the deflection is larger.

If he finds an average deflection of 0.0129 in his sample, is there enough evidence to reject H_0 ? and a.s.d. of .003
t-test (one sample mean t-test)

$$H_0: \mu = .012$$

$$H_a: \mu > .012$$



$$t = \frac{.0129 - .012}{.003 / \sqrt{10}}$$

```
> (.0129-.012)/(.003/sqrt(10))
[1] 0.9486833
```

$$\alpha = .05 \rightarrow q_t(.95, 9)$$

Based on $\alpha = .05$, we will fail to reject the claim that the mean deflection of the steel beams is .012 in.

p-value

$$\left(\begin{array}{l} > 1 - \text{pt}(.9487, 9) \\ [1] 0.183775 \\ > \alpha \end{array} \right)$$

~~Friday~~

It is believed that 35% of all voters favor a particular candidate. How large of a SRS is required so that the margin of error of the estimate of percentage of all voters in favor is no more than 3% at the 95% confidence level?

Popper 22

1. The width of a confidence interval is dependent on the level of confidence.

☒ a. True

b. False

2. A 95% confidence level means that if we take 100 samples and compute confidence intervals for each, approximately 95 of the 100 CIs will contain the true population parameter.

☒ a. True

b. False

3. A confidence interval is centered around the population parameter.

☒ a. True

b. False

4. A 95% confidence interval for a proportion has a critical value , z^* , of

- a. 1.645 ← 90%
- ☒ b. 1.960
- c. 2.170
- d. 0.975
- e. none of these



5. Before we can make an inference about a population, certain conditions must be satisfied. These conditions are dependent on the type of confidence interval we are finding.

- ☒ a. True
- b. False