Math 3339

Section 27204
MWF 10-11:00am AAAud 2

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Office Hours:
M & Th noon – 1:00 pm & T 1:00 – 2:00 pm
and by appointment

Anova

Another example:

The data in the accompanying table resulted from an experiment run in a completely randomized design in which each of four treatments was replicated five times.

5

$N = 4 \cdot 5 = 20$

						Total	Mean
Group 1	6.9	5.4	5.8	4.6	4.0	26.70	5.34
Group 2	8.3	6.8	7.8	9.2	6.5	38.60	7.72
Group 3	8.0	10.5	8.1	6.9	9.3	42.80	8.56
Group 4	5.8	3.8	6.1	5.6	6.2	27.50	5.50
All Groups						135.60	6.78

$m = 4$

← grand mean $\bar{x}_{..}$

$df = \underset{m-1}{3}, \underset{N-m}{16}$

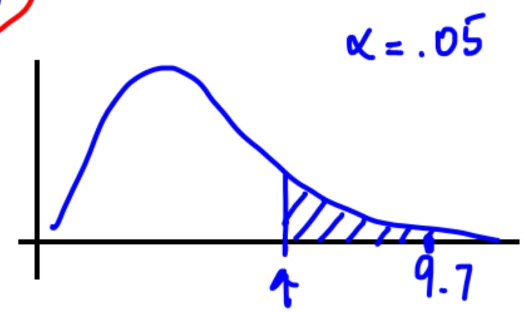
$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a: at least two groups have different means (significantly)

Part of the resulting ANOVA table is

Source	SS	DF	MS	F	Pvalue
Treatments	38.820	3	12.940	$12.94/1.331 = 9.722$	
Error	21.292	16	1.331		

Complete the ANOVA table.



test statistic $f = 9.722$

pvalue $P(f > 9.722)$

$1 - pf(9.722, 3, 16) = .000685 < .01$

```
> qf(.95,3,16)
[1] 3.238872
```

Perform a significance test to see if at least two of the μ_i's are different.

Based on $\alpha = .05$ (or even $\alpha = .01$) we will reject the null hyp and conclude at least two of the groups have sign. different means.

Tukey's Method (The *T* Method)

The T-method is used to determine *which* pair (or pairs) of means differ significantly.

Tukey's Procedure (the *T* Method)

from Tukey distribution

The *T* Method for Identifying Significantly Different μ_i's

Select α, extract $Q_{\alpha,I,I(J-1)}$ from ~~Appendix Table A.10~~, and calculate $w = Q_{\alpha,I,I(J-1)} \cdot \sqrt{\text{MSE}/J}$. Then list the sample means in increasing order and underline those pairs that differ by less than w. Any pair of sample means not underscored by the same line corresponds to a pair of population or treatment means that are judged significantly different.

w = width

$$= \text{qtukey}(1-\alpha, m, N-m) \cdot \sqrt{\frac{MSE}{\#\text{treatments per group}}}$$

Let's use this method on previous example to determine which pairs differ significantly:

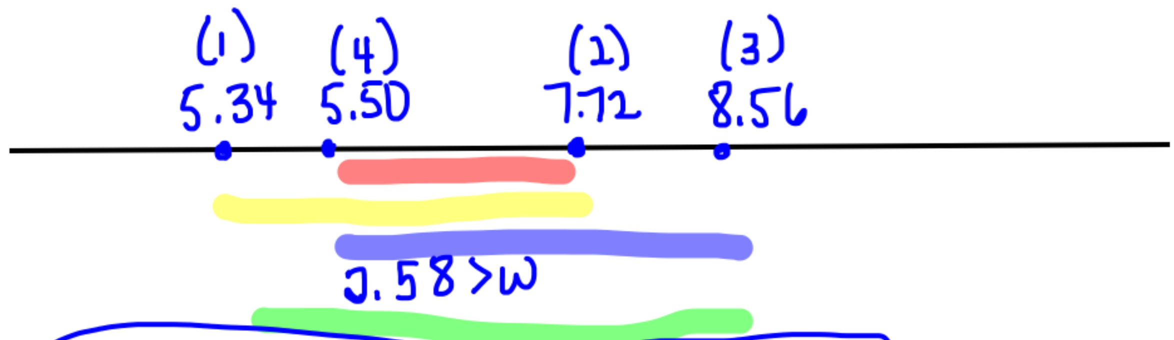
$$W = \text{qtukey}(.95, 4, 16)\sqrt{\frac{1.331}{5}} = 2.0876$$

$\bar{X}_1 = 5.34$

$\bar{X}_2 = 7.72$

$\bar{X}_3 = 8.56$

$\bar{X}_4 = 5.50$



$\boxed{(1,2)\ (1,3)\ (4,2)\ (4,3)}$

Pairwise *t* – test

An alternative to Tukey's method (and the most common method used) is to do a t-test for each pair of means.

There is an issue with this though, our P(Type 1 error) becomes greater than α for multiple comparisons. So we need to do a method of adjustments to reduce the significance level of the pairwise test enough so that the probability of one or more type I errors in the whole set of comparisons is less than α.

The **Bonferroni Method** uses α/k instead of α for the testing of *k* comparisons.

R code:
>pairwise.t.test(*data*,*categories*,"bonferroni")

Let's revisit this example from Wednesday:
Example: An experiment was conducted to measure and compare the effectiveness of various feed supplements on the growth rate of chickens. The data is in the chickwts data package.
Open R Studio, make sure the package datasets is checked.
Type: chickwts to see the data.
Now let's make the model:

```
> chick.lm=lm(weight~feed,data = chickwts)
> summary(chick.lm)
> anova(chick.lm)
> pairwise.t.test(chickwts$weight,chickwts$feed,"bonferroni")
```

The adjustment methods include the Bonferroni correction ("bonferroni") in which the p-values are multiplied by the number of comparisons

⇒ Compare to α

```
        Pairwise comparisons using t tests with pooled SD

data:  chickwts$weight and chickwts$feed

          casein  horsebean linseed meatmeal soybean
horsebean 3.1e-08 -         -       -        -
linseed   0.00022 0.22833   -       -        -
meatmeal  0.68350 0.00011   0.20218 -        -
soybean   0.00998 0.00487   1.00000 1.00000  -
sunflower 1.00000 1.2e-08   9.3e-05 0.39653  0.00447

P value adjustment method: bonferroni
```

pairs whose means differ significantly are
casein and horsebean
casein and linseed
meatmeal and horsebean
soybean and casein
soybean and horsebean
sunflower and horsebean
sunflower and linseed
sunflower and soybean

two

Inferences Concerning a Difference Between Population Proportions

Confidence Intervals:

$$\left(\widehat{p_1}-\widehat{p_2}\right) \pm z^* \sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1}+\frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$$

qnorm

Tests of two population proportions:

The rejection region for a hypothesis testing of two population proportions:

$\pi_1 = \pi_2$

$$H_0 : p_1 = p_2 \ \ (or\ p_1 - p_2 = \delta)$$

$$H_a : p_1 \neq p_2 \ or\ p_1 < p_2 \ or\ p_2 > p_2$$

$$z = \frac{\widehat{p_1}-\widehat{p_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad where\ \hat{p} = \frac{X+Y}{n_1+n_2}$$

← pooled proportion

test stat

Assumptions:

1. $n_1p_1 \geq 10, n_1(1-p_1) \geq 10, n_2p_2 \geq 10, n_2(1-p_2) \geq 10$

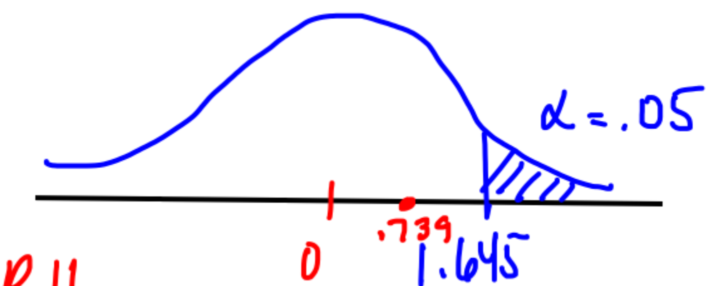
2. two independent samples

Example: If $n_1 = n_2 = 100$, $y_1 = 67$, $y_2 = 62$ test the claim that $p_1 > p_2$ using $\alpha = 0.05$ significance level.

$H_0: p_1 = p_2$

$H_a: p_1 > p_2$

$\hat{p}_1 = \frac{67}{100} = .67$ $\quad \hat{p}_2 = \frac{62}{100}$ $\quad \hat{p} = \frac{67+62}{100+100} = \frac{129}{200}$



$$z = \frac{.67 - .62}{\sqrt{\frac{129}{200}\left(\frac{71}{200}\right)\left(\frac{1}{100} + \frac{1}{100}\right)}} = .739$$

FRH$_0$

pvalue: $1 - \text{pnorm}(.739) = .23 > \alpha$

Example:

The authors of the article "Adjuvant Radiotherapy and Chemotherapy in Node-Positive Premenopausal Women with Breast Cancer" (*New Engl. J. of Med.*, 1997: 956–962) reported on the results of an experiment designed to compare treating cancer patients with chemotherapy only to treatment with a combination of chemotherapy and radiation. Of the 154 individuals who received the chemotherapy-only treatment, 76 survived at least 15 years, whereas 98 of the 164 patients who received the hybrid treatment survived at least that long. With p_1 denoting the proportion of all such women who, when treated with just chemotherapy, survive at least 15 years and p_2 denoting the analogous proportion for the hybrid treatment, $\hat{p}_1 = 76/154 = .494$ and $98/164 = .598$. A confidence interval for the difference between proportions based on the traditional formula with a confidence level of approximately 99% is

$\hat{p}_1 = .494$

$\hat{p}_2 = .598$

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(.494 - .598) \pm 2.576 \sqrt{\frac{.494(.506)}{154} + \frac{.598(.402)}{164}}$$

$$(-.247, .039)$$

$H_0: p_1 = p_2 \ni p_1 - p_2 = 0$

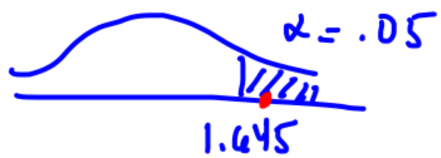
90% CI: $(-.195, -.013)$

A random sample of 200 freshmen and 100 seniors at Upper Wabash Tech are asked whether they agree with a plan to limit enrollment in crowded majors as a way of keeping the quality of instruction high. Of the students sampled, 160 freshmen and 70 seniors opposed the plan. We want to determine if there is a greater proportion of freshmen who oppose the plan compared to the proportion of seniors who oppose it.

$H_0: P_F = P_s$

$H_a: P_F > P_s$

$\hat{P}_F = \frac{160}{200} = .8$ $\qquad \hat{P}_s = \frac{70}{100} = .7$



$\hat{P} = \frac{160+70}{300}$

$$z = \frac{.8 - .7}{\sqrt{\frac{230}{300}\left(\frac{70}{300}\right)\left(\frac{1}{200}+\frac{1}{100}\right)}} = 1.93$$

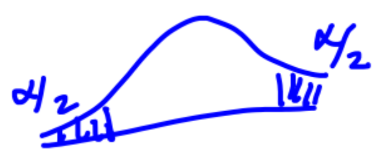
RH_0

pvalue $1 - \text{pnorm}(1.93) = .03 < \alpha$

Find a 95% confidence interval for the difference between those two proportions

$$(.8 - .7) \pm z^* \sqrt{\frac{.8(.2)}{200} + \frac{.7(.3)}{100}} = (-.0055, .2055)$$

$z^* = 1.96$



$H_a: P_F \neq P_s$

Popper 28

1. A Statistics teacher asks her class to construct confidence intervals given a sample mean of .45 and a variety of sample sizes. Four groups of students are each given a different sample size and the intervals from the groups are displayed on the board. Which of the following intervals must be incorrect?
   a. Group A: (.37, .53)
   b. Group B: (.41, .49)
   c. Group C: (.35, .55)
   d. Group D: (.40, .49)
   e. All are correct

2. It has always been thought that of the three major radio stations that service a large metropolitan area, station A had a 30% share of the market while stations B enjoyed a 35% market share. A study of 75 randomly chosen radio listeners indicated that 30 people listen to station A, 45 listen to station B. Which of the following tests would be most appropriate in establishing that the market shares have changed?
   A. t-test of means
   B. two-sample proportion z-test
   C. matched pair t-test
   D. ~~two-sample proportion t-test~~

3. One of your peers claims that boys do better in math classes than girls. Together you run two independent simple random samples and calculate the given summary statistics of the boys and the girls for comparable math classes. In Calculus, 15 boys had a mean percentage of 82.3 with standard deviation of 5.6 while 12 girls had a mean percentage of 81.2 with standard deviation of 6.7. Which of the following would be the most appropriate test for establishing whether boys do better in math classes than girls?

A. ANOVA test
B. two-sample t-test for means
C. two-sample z-test for proportions
D. none of these tests would be appropriate

4. Suppose we have two SRSs from two distinct populations and the samples are independent. We measure the same variable for both samples. Suppose both populations of the values of these variables are normally distributed but the means and standard deviations are unknown. For purposes of comparing the two means, we use

   a. Two-sample t procedures
   b. Matched pairs t procedures
   c. z procedures
   d. none of these

5. A