CUIN 7333

Notes: Sequences (part III) and Series

Special Sequences:

Arithmetic: An arithmetic sequence is a sequence in which the common difference between successive terms, *d,* is constant.

$$a_{n+1} = a_n + d$$

Geometric: A geometric sequence is a sequence in which the common ratio between successive terms, *r,* is constant.

$$a_{n+1} = a_n r$$

Examples:

Determine whether the indicated sequence can be the first three terms of an arithmetic or geometric sequence, and, if so, find the common difference or common ratio and the general term a_n.

24. $1, 4, 9, \ldots$ not arithmetic & not geometric $\frac{4}{1} \neq \frac{9}{4}$

$3 \quad 5$ ←

25. $2, -4, 8, \ldots$ Geometric $\frac{-4}{2} = \frac{8}{-4} \Rightarrow r = -2$

27. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \ldots$ Geometric $r = \frac{1}{3}$

Finding terms:

Arithmetic: $a_n = a + (n-1)d$.

Geometric: $a_n = a \cdot r^{n-1}$

Examples:

Find the 10th term in each sequence:

A 32. 11, 9, 7, . . . $a = a_1 = 11$ $n = 10$ $d = -2$ $a_{10} = 11 + (10-1)(-2)$

$= -7$

G 40. 2, 6, 18, . . . $a_1 = 2$ $r = 3$ $n = 10$

$a_{10} = 2 \cdot 3^{10-1} = 2 \cdot 3^9$

Series:

Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ be a given sequence. Suppose we are asked to add up all the terms of the sequence. That is, suppose we are asked to calculate

$$a_1 + a_2 + a_3 + \ldots + a_n + \ldots .$$

The sum as a sequence of partial sums:

Let
$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= a_1 + a_2 + a_3 \\ S_4 &= a_1 + a_2 + a_3 + a_4 \\ &\vdots \\ S_n &= a_1 + a_2 + a_3 + \ldots + a_n \\ &\vdots \end{aligned}$$

This new sequence is called the *sequence of partial sums* of the given sequence a_n.

Finite Sums:

Arithmetic:

$$S_n = \frac{n}{2}(a + a_n) \quad \text{or} \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric:

$$S_n = \frac{a - ar^n}{1 - r} \quad \text{provided } r \neq 1$$

Infinite Geometric Series:

$$S = \frac{a}{1-r}, \quad |r| < 1$$

Examples:

Let $a_1, a_2, a_3, \ldots$ be an arithmetic sequence. Find the indicated quantities.

35. $a_1 = a = 7$, $d = 2$; find a_5 and S_{10}.

$$a_5 = 7 + (5-1)(2) = 15$$

$$S_{10} = \frac{10}{2}\left[2(7) + (10-1)(2)\right] = 160$$

Let $a_1, a_2, a_3, \ldots$ be a geometric sequence. Find the indicated quantities.

43. $a_1 = a = 3$, $r = -2$; find a_6 and S_6.

$$a_6 = (3)(-2)^{6-1} = 3(-2)^5 = -96$$

$$S_6 = \frac{3 - 3(-2)^6}{1-(-2)} = -63$$

Determine whether the geometric series has a finite sum. If it does, find it.

47. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

$$r = \frac{-1}{2}$$

Since $|r| = \left|-\frac{1}{2}\right| < 1$ then this series has a sum

$$S = \frac{a}{1-r} = \frac{1}{1-(-1/2)} = \frac{2}{3}$$