CUIN 7333

Notes: Sequences (part II)

More about recursive functions. Recall that we said a recursion formula is a formula that gives a_{n+1} in terms of one or more of the terms that precede a_{n+1} .

Examples:

$$a_{1} = 1 \text{ and } a_{n+1} = (n+1)a_{n}$$

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$$a_{2} = a_{n+1} \text{ so } a_{2} = n+1 \text{ and } n=1$$

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Q3 = Qn+1 50 3= n+1 => n=2

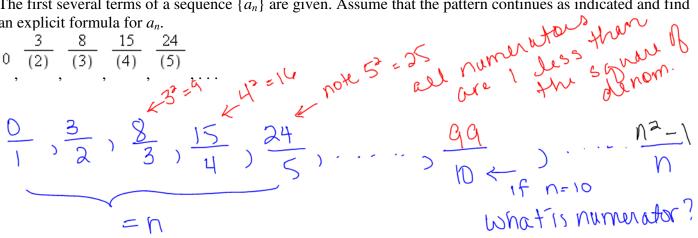
This example represents the <u>factorial</u> function

$$a_1 = a_2 = 1$$
 and $a_{n+2} = a_n + a_{n+1}$

This second example is the <u>Fibonacci</u> sequence. It can actually be shown that $a_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n \sqrt{5}}$ and that if we found a sequence of ratios of consecutive terms $\left(\frac{a_{n+1}}{a_n}\right)$, the values would approach $\frac{2}{\sqrt{5}-1}$, the *golden ratio* (ϕ =1.61803398875...). For more information on the golden ratio see here: <u>http://www.jimloy.com/geometry/golden.htm</u>

Finding a formula for a sequence:

The first several terms of a sequence $\{a_n\}$ are given. Assume that the pattern continues as indicated and find an explicit formula for a_n .



Sequences can be bounded or unbounded.

I have both a upper AND lower bound

The sequence $\{an\}$ is said to be

- *increasing* if an < an+1 for all n, •
- *nondecreasing* if $an \leq an+1$ for all n,
- *decreasing* if an > an+1 for all n,
- *nonincreasing* if $an \ge an+1$ for all n. •

A sequence that satisfies any of these conditions is called *monotonic*.

Example: Determine the boundedness and monotonicity of the sequence with a_n as indicated.

$$\frac{(n+8)^2}{n^2}$$

Write out first few terms:

$$\frac{q^{2}}{1^{2}}, \frac{10^{2}}{2^{2}}, \frac{11^{2}}{3^{2}}, \dots, \frac{108^{2}}{100^{2}}, \dots, \frac{10008^{2}}{1000^{2}}, \dots, \frac{10008^{2}}{10000^{2}}, \dots$$
9, 25, $\frac{121}{9}, \dots, \frac{10000^{2}}{9}, \dots, \frac{10000^{2}}{1000^{2}}, \dots, \frac{10000^{2}}{10000^{2}}, \dots$
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4, 25, $\frac{121}{9}, \dots, \frac{10000^{2}}{9}, \dots, \frac{10000^{2}}{1000^{2}}, \dots, \frac{10000^{2}}{10000^{2}}, \dots, \frac{10000^{2}}{10000^{2}}, \dots$
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We are concerned with limits of sequences as n approaches infinity.

A sequence that has a limit is said to be *convergent*. A sequence that has no limit is said to be *divergent*.

Every convergent sequence is bounded and every unbounded sequence is divergent

THM - A bounded, nondecreasing sequence converges to its least upper bound; a bounded, nonincreasing sequence converges to its greatest lower bound.

Examples:

1.
$$\left\{\frac{1}{n^2}\right\} = 1, \frac{1}{4}, \frac{1}{9}, \frac{1}{10}, \frac{1}{25}, \dots$$

decreasing
upper bound =1 bounded below by D
limit as $n \Rightarrow \infty$ is O
2. $\left\{\sqrt{4-\frac{1}{n}}\right\} = \sqrt{3}, \sqrt{3^{1/2}}, \sqrt{3^{2/3}}, \sqrt{3^{2/3}}, \sqrt{3^{2/3}}, \frac{\sqrt{3^{2/3}}}{\sqrt{3^{2/3}}}, \frac{\sqrt{3^{2/3}}}{\sqrt{3^{2/3}}$

Visualizing limits with Geogebra (free graphing software). <u>http://www.geogebra.org/cms/</u>