Factoring Binomials

I. Difference of two squares.

\[ a^2 - b^2 = (a - b)(a + b) \]

Examples:

1. \( d^2 - 16 \) 
2. \( -25 + m^4 \)
3. \( 36x^6 - 16y^{10} \) 
4. \( (x - 2)^2 - 36 \)
5. \( 4n^2 + 25m^2 \)

II. Sum or Difference of two cubes.

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

Examples:

6. \( x^3 + 8 \) 
7. \( 27x^3 - 1 \)
8. \( 64x^3y^6 + 125z^9 \) 
9. \( 8x^8 - y^{15} \)
Factoring by Grouping

Steps:
1. Take out GCF.
2. Look for perfect square trinomial.
3. Group first 2 and second 2 together.
4. Take out GCF of each group.
5. Factor it out.

Examples:

1. $3a^2 + 15a + xa + 5x$
2. $2mn - 4n^3 + mp - 2n^2p$

3. $12bc + 4c + 6b^2 + 2b$
4. $6ab - 3b + 2a - 1$

5. $8x^2y + 4xy - 6xy^2 - 3y^2$
6. $4xa + 3x^2b - 2xb - 6x^2a$

7. $30x^2m + ny - 6my - 5nx^2$
8. $x^2 + 4x + 4 - y^2$
Factoring Trinomials

Rules:
1. Take out GCF first!
2. Always look at second sign
   If second sign is a ________, both signs are whatever the first sign is.
   
   \[x^2 + bx + c = (\_ + \_)(\_ + \_)]
   
   \[x^2 - bx + c = (\_ - \_)(\_ - \_)]
   
   If second sign is a ________, both signs are different.
   
   \[x^2 + bx - c = (\_ + \_)(\_ - \_)]
   
   \[x^2 - bx - c = (\_ + \_)(\_ - \_)]

3. If the two signs are the same, the last term will "add" to make the middle term. If the two signs are different, the last term will "subtract" to make the middle term.

Examples:

1. \[4x^2 + 8x + 3\]
2. \[2x^2 - 13x + 15\]

3. \[3x^2 + 17x - 28\]
4. \[10x^2 - 39x - 27\]

5. \[6x^2 + 9x + 3\]
6. \[25x^2 + 20x + 4\]

7. \[10x^2y - 34xy + 28y\]
8. \[12x^4 + 30x^2y^2 + 12y^4\]
This is the process you should be going through when factoring any problem.

**FACTORING**

1. Look for the GCF

2. If you have a trinomial...
   - 3. If possible, factor into 2 binomials.
     - 4. Check the binomials, are they the difference of squares or sum/difference of cubes?

2. If you have a binomial...
   - 3. Check for the difference of squares.

2. If you have 4 or more terms...
   - 3. Group if possible.
   - 4. Look for the GCF in the groups.
     - 5. If the GCF is a binomial, look and see if it is the difference of squares or sum/difference of cubes.