CUIN 7333

Notes: Sequences (part II)

More about recursive functions. Recall that we said a recursion formula is a formula that gives a_{n+1} in terms of one or more of the terms that precede a_{n+1}.

Examples:

$$a_1 = 1 \quad \text{and} \quad a_{n+1} = (n+1)a_n$$

This example represents the ____

$$a_1 = a_2 = 1 \quad \text{and} \quad a_{n+2} = a_n + a_{n+1}$$

This second example is the _____________________________________ sequence. It can actually be shown that $a_n = \dfrac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n\sqrt{5}}$ and that if we found a sequence of ratios of consecutive terms $\left(\dfrac{a_{n+1}}{a_n}\right)$, the values would approach $\dfrac{2}{\sqrt{5}-1}$, the *golden ratio* ($\phi = 1.61803398875...$). For more information on the golden ratio see here: http://www.jimloy.com/geometry/golden.htm

Finding a formula for a sequence:

The first several terms of a sequence $\{a_n\}$ are given. Assume that the pattern continues as indicated and find an explicit formula for a_n.

$$0, \frac{3}{(2)}, \frac{8}{(3)}, \frac{15}{(4)}, \frac{24}{(5)}, \ldots$$

Sequences can be bounded or unbounded.

The sequence $\{an\}$ is said to be

- *increasing* if $an < an+1$ for all n,
- *nondecreasing* if $an \leq an+1$ for all n,
- *decreasing* if $an > an+1$ for all n,
- *nonincreasing* if $an \geq an+1$ for all n.

A sequence that satisfies any of these conditions is called *monotonic*.

Example: Determine the boundedness and monotonicity of the sequence with a_n as indicated.

$$\frac{(n+8)^2}{n^2}$$

We are concerned with limits of sequences as n approaches infinity.
A sequence that has a limit is said to be *convergent*. A sequence that has no limit is said to be *divergent.*

Every convergent sequence is bounded and every unbounded sequence is divergent

THM - A bounded, nondecreasing sequence converges to its least upper bound; a bounded, nonincreasing sequence converges to its greatest lower bound.

Examples:

1. $\left\{\dfrac{1}{n^2}\right\}$

2. $\left\{\sqrt{4-\dfrac{1}{n}}\right\}$

3. $\left\{\dfrac{n^2}{n+1}\right\}$

Visualizing limits with Geogebra (free graphing software). http://www.geogebra.org/cms/