What we have learned so far:

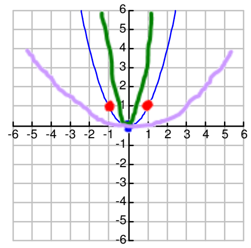
. Linear Functions
. Absolute Value Functions

Now lets investigate transformations and graphs of:

. Quadratic Functions
. Square Root Functions
. Cubic Functions
. Rational Functions

The Quadratic Function

The **quadratic function** is another parent function. The equation for the quadratic function is $\boxed{y = x^2}$ and its graph is a bowl-shaped curve called a **parabola**. The point $(0,0)$ is called the vertex.



The **vertex form** for all quadratics is $y = a(x-h)^2 + k$, and follows all the same rules for determining translations on the parent function except the slope. Notice the coefficient is in front of the squared term.

If $|a| = 1$, the parabola is standard size and 2 points are graphed up 1 and over 1 on each side of the vertex.

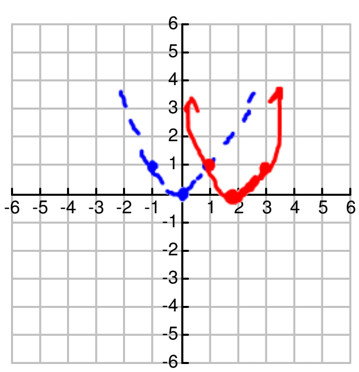
If $|a| > 1$, the parabola is skinnier which represents a vertical stretch. The graph is drawn between the basic points.

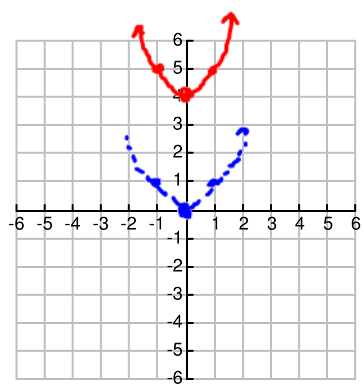
If $0 < |a| < 1$, the parabola is wider which represents a vertical compression. The graph is drawn outside of the basic points.

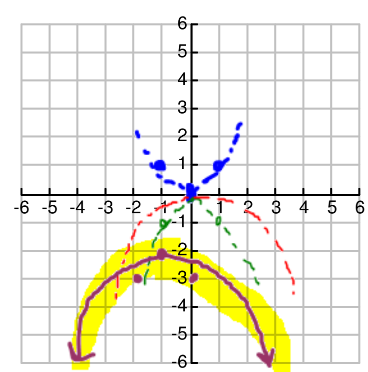
Example 1. For each problem, write the equation in the vertex form $y = a(x-h)^2 + k$.

a) state the parent function
b) name the function
c) is there a reflection ← $a<0$
d) state the vertical stretch or compression
e) state the phase (or horizontal) shift
f) state the vertical shift

	a. $y = 1(x-2)^2 + 0$	b. $y = x^2 + 4 = 1(x-0)^2 + 4$	c. $y = -\frac{1}{4}(x+1)^2 - 2$
a)	$y = x^2$	$y = x^2$	$y = x^2$
b)	quadratics	quadratic	quadratic
c)	no	no	yes $(a<0)$
d)	none $(a=1)$	none $(a=1)$	compression $\lvert a\rvert = \frac{1}{4}$
e)	2 right	0	1 left
f)	0	up 4	2 down

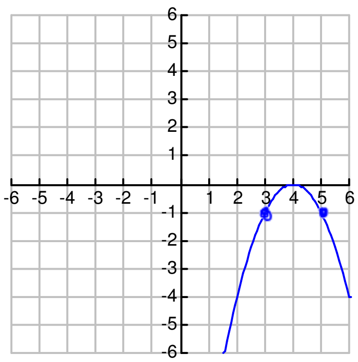






Example 2. Write the equation of each parabola from the graph and the given information.

4 right reflection ($a<0$)
0 v.s. $|a| = 1$

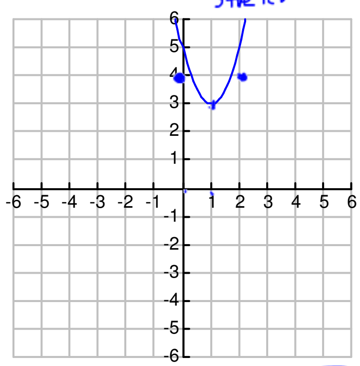


$y = -1(x-4)^2 + 0$

$\boxed{y = -(x-4)^2}$

a=2 or a= $\frac{1}{2}$

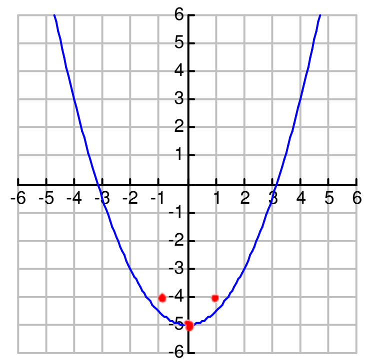
no refl.
1 right, 3 up
stretch



$\boxed{y = 2(x-1)^2 + 3}$

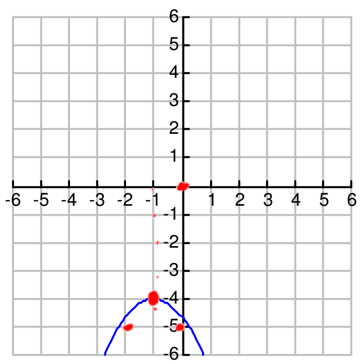
$\boxed{a = \frac{2}{3}}$ or a=3

down 5 no p.s.
compression
no reflect.



$y = \frac{2}{3}(x-0)^2 - 5$

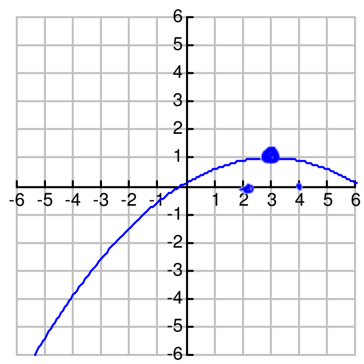
$\boxed{y = \frac{2}{3}x^2 - 5}$



a=4 or $\boxed{a = \frac{1}{4}}$

1 left 4 down
reflection

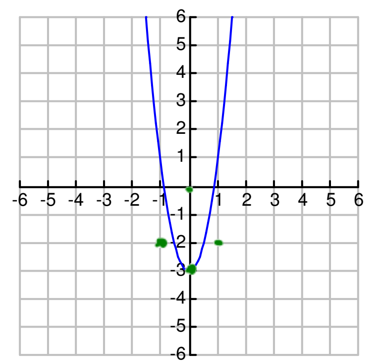
$\boxed{y = -\frac{1}{4}(x+1)^2 - 4}$



a=10 or $\boxed{a = \frac{1}{10}}$

3 right 1 up
reflection

$\boxed{y = -\frac{1}{10}(x-3)^2 + 1}$



a=.2 or $\boxed{a=5}$

down 3 no p.s.
stretch
no reflection

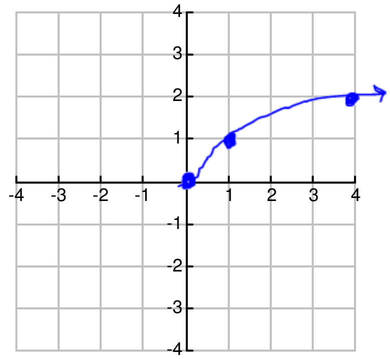
$y = 5(x-0)^2 - 3$

$\boxed{y = 5x^2 - 3}$

The Square Root Function

The **square root function** is another parent function. The equation of the square root function is $\boxed{y = \sqrt{x}}$.
Fill in the chart of ordered pairs and look at the graph.

x	0	1	4
y	0	1	2



The graph should be a smooth curve that looks like half of a parabola.

What is the domain? $x \geq 0$

What is the range? $y \geq 0$

To determine the domain of a square root function without graphing, set the expression under the radical sign greater than or equal to zero. (The number under the square root must be 0 or a positive value.)

Example 1: Find the domain for the function $y = \sqrt{2x+3}$.
Write the answer in interval notation.

$$2x + 3 \geq 0$$
$$2x \geq -3$$
$$x \geq -\frac{3}{2}$$

Answer: $\left[-\frac{3}{2}, \infty\right)$

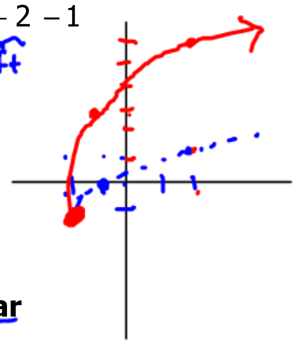
Example 2: Find the domain for the function $y = 3\sqrt{4x-5} - 1$.

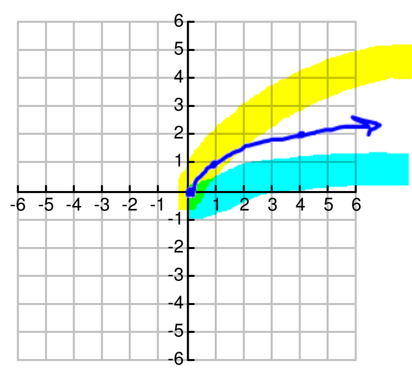
$4x - 5 \geq 0$
$4x \geq 5$
$x \geq \frac{5}{4}$
$\left[\frac{5}{4}, \infty\right)$

The graphing form for all square root functions is $y = a\sqrt{x-h} + k$. If $a < 0$, the graph is reflected across the x-axis. (a flip) The **value** of **a** will determine the vertical stretch or compression. The translations are determined by h and k. Each point on the parent function moves horizontally h units and vertically k units.

Example 3: no reflection Graph $y = 3\sqrt{x+2} - 1$ stretch 1 down 2 left

Graph the parent function.
Each point on the parent function is moved horizontally to the left 2 units and vertically down 1 unit. The graph stays above the translated horizontal axis since $a > 0$.
Since the value of a is 3, each point on the parent function is **3 times as far** from the translated horizontal axis.



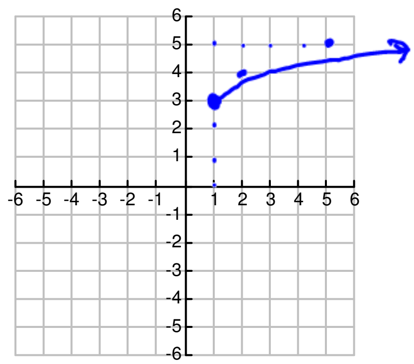


Graph the new function. State the domain and range in interval notation.

D = $[-2, \infty)$

R = $[-1, \infty)$

Example 4: $y = \frac{1}{2}\sqrt{x-1} + 3$



D = $[1, \infty)$

R = $[3, \infty)$

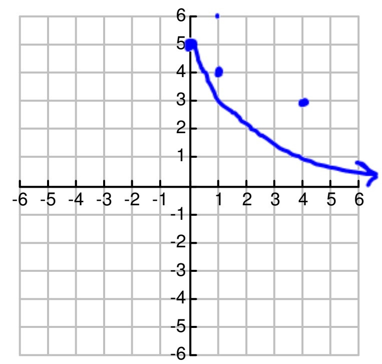
Example 5: $y = -4\sqrt{x} + 5$

stretch

0 p s

up 5

reflection

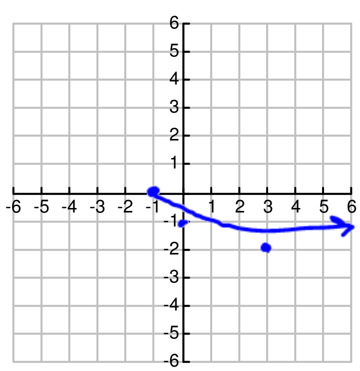


D = $[0, \infty)$

R = $(-\infty, 5]$

Example 6: $y = -\frac{1}{4}\sqrt{x+1}$

1 left 0 up

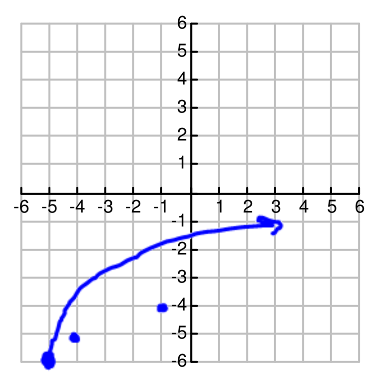


D = $[-1, \infty)$

R = $(-\infty, 0]$

Example 7: $y = 3\sqrt{x+5} - 6$

left 5 down 6

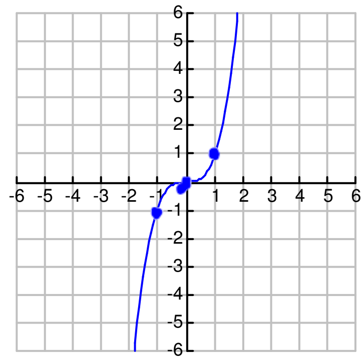


D = $[-5, \infty)$

R = $[-6, \infty)$

The Cubic Function

The **cubic function** is a parent function with the equation $y = x^3$. The graph is shown below.



The translations are performed the same way as the other functions using the equation $\boxed{y = m(x-h)^3 + k}$.

For each example explain the translations on the parent function to obtain the following graph.

Example 1. $y = 2(x-3)^3 + 1$

Reflection no

Stretch or compression stretch of 2

Phase shift 3 right

Vertical shift up 1

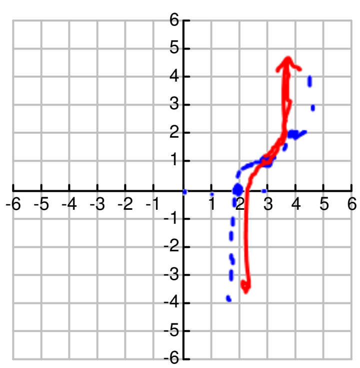
Example 2. $y = -\frac{1}{3}(x+2)^3 - 4$

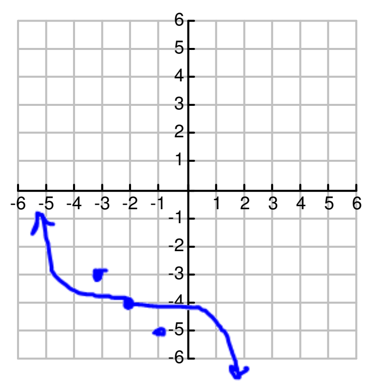
Reflection yes

Stretch or compression Compr. 1/3

Phase shift 2 left

Vertical shift down 4

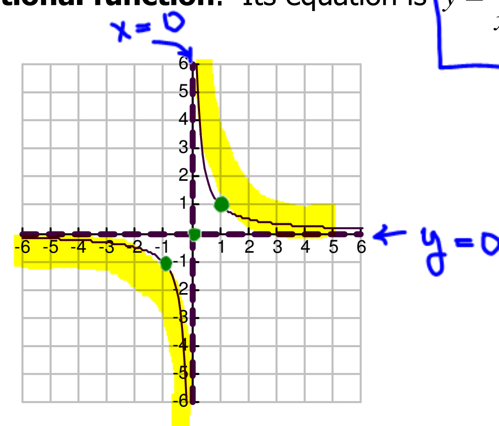




The Rational Function

Another parent function is called the **rational function**. Its equation is $\boxed{y = \frac{1}{x}}$.

Here is its graph:



There is a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.
Instead of graphing rational functions using vertical and horizontal asymptotes, we will look at the rational functions as a family of the parent function. We will use reflections, phase shifts and vertical shifts to graph the family of rational functions.

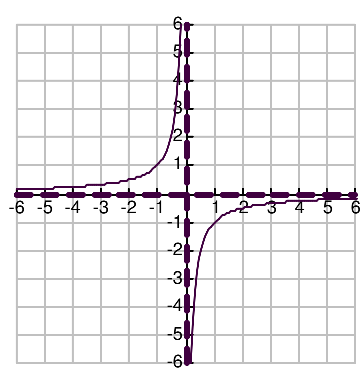
The general equation for all rational functions is:

$$\boxed{y = \frac{a}{x-h} + k}$$

where the sign of **a** determines a **reflection, h** determines the **phase shift** for the **vertical asymptote** and **k** determines the **vertical shift** for the **horizontal asymptote**. In this lesson we will not be concerned with finding exact values for the x and y intercepts. Our graph will be a rough sketch of the function.

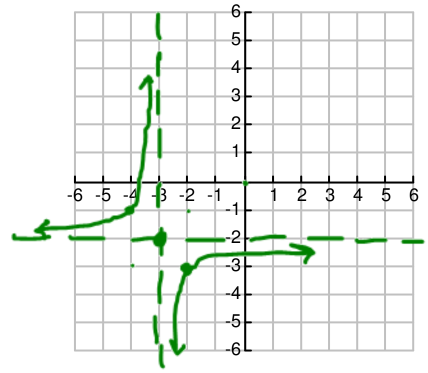
Example 1: $y = \frac{-1}{x}$

a = –1 represents a reflection (the graph starts below the x-axis, starting with the positive side)
phase shift = 0
vertical shift = 0
Graph:



Example 2 : $y = -\frac{1}{x+3} - 2$

Graph:



a = -1 (reflection)

phase shift = 3 left

vertical shift = 2 down

Example 3: Write the equation from the graph in the form $y = \frac{a}{x-h} + k$

reflection ? yes

phase shift 1 left

vertical shift 4 up

$$y = \frac{-1}{x+1} + 4$$

Graph: