The Quadratic Function

The quadratic function is another parent function. The equation for the quadratic function is \( y = x^2 \) and its graph is a bowl-shaped curve called a parabola. The point \((0, 0)\) is called the vertex.

The vertex form for all quadratics is \( y = a(x - h)^2 + k \), and follows all the same rules for determining translations on the parent function except the slope. Notice the coefficient is in front of the squared term.

- If \( a = 1 \), the parabola is standard size and 2 points are graphed up 1 and over 1 on each side of the vertex.
- If \( a > 1 \), the parabola is skinnier which represents a vertical stretch. The graph is drawn between the basic points.
- If \( 0 < a < 1 \), the parabola is wider which represents a vertical compression. The graph is drawn outside of the basic points.

Example 1. For each problem, write the equation in the vertex form \( y = a(x - h)^2 + k \).

<table>
<thead>
<tr>
<th></th>
<th>a) state the parent function</th>
<th>d) state the vertical stretch or compression</th>
<th>b) name the function</th>
<th>e) state the phase (or horizontal) shift</th>
<th>c) is there a reflection</th>
<th>f) state the vertical shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( y = (x - 2)^2 )</td>
<td></td>
<td>( y = x^2 + 4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>( y = x^2 + 4 )</td>
<td></td>
<td>( y = -\frac{1}{4}(x + 1)^2 - 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) ________________ a) ________________ a) ________________  
b) ________________ b) ________________ b) ________________  
c) ________________ c) ________________ c) ________________  
d) ________________ d) ________________ d) ________________  
e) ________________ e) ________________ e) ________________  
f) ________________ f) ________________ f) ________________
Example 2. Write the equation of each parabola from the graph and the given information.

\[ a = 2 \text{ or } a = \frac{1}{2} \quad a = \frac{2}{3} \text{ or } a = 3 \]

\[ a = 4 \text{ or } a = \frac{1}{4} \quad a = 10 \text{ or } a = \frac{1}{10} \quad a = .2 \text{ or } a = 5 \]
The Square Root Function

The square root function is another parent function. The equation of the square root function is \( y = \sqrt{x} \).

Fill in the chart of ordered pairs and look at the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The graph should be a smooth curve that looks like half of a parabola.

What is the domain? _____________________

What is the range? ______________________

To determine the domain of a square root function without graphing, set the expression under the radical sign greater than or equal to zero. (The number under the square root must be 0 or a positive value.)

**Example 1:** Find the domain for the function \( y = 2x + 3 \).

Write the answer in interval notation.

\[
2x + 3 \geq 0 \\
2x \geq -3 \\
x \geq -\frac{3}{2}
\]

Answer: \( \left[ -\frac{3}{2}, \infty \right) \)

**Example 2:** Find the domain for the function \( y = 3\sqrt{4x - 5} - 1 \).

The graphing form for all square root functions is \( y = a\sqrt{x - h} + k \). If \( a < 0 \), the graph is reflected across the \( x \)-axis. (a flip) The value of \( a \) will determine the vertical stretch or compression. The translations are determined by \( h \) and \( k \). Each point on the parent function moves horizontally \( h \) units and vertically \( k \) units.

**Example 3:** Graph \( y = 3\sqrt{x + 2} - 1 \)

Graph the parent function.
Each point on the parent function is moved horizontally to the left 2 units and vertically down 1 unit. The graph stays above the translated horizontal axis since \( a > 0 \).
Since the value of \( a \) is 3, each point on the parent function is 3 times as far from the translated horizontal axis.

Graph the new function. State the domain and range in interval notation.

\[ D = \quad R = \]
Example 4: \( y = \frac{1}{2}\sqrt{x - 1} + 3 \)

D = 

R = 

Example 5: \( y = -4\sqrt{x} + 5 \)

D = 

R = 

Example 6: \( y = -\frac{1}{4}\sqrt{x + 1} \)

D = 

R = 

Example 7: \( y = 3\sqrt{x + 5} - 6 \)

D = 

R = 
The Cubic Function

The cubic function is a parent function with the equation \( y = x^3 \). The graph is shown below.

The translations are performed the same way as the other functions using the equation \( y = m(x - h)^3 + k \).

For each example explain the translations on the parent function to obtain the following graph.

Example 1. \( y = 2(x - 3)^3 + 1 \)  
Example 2. \( y = -\frac{1}{3}(x + 2)^3 - 4 \)

Reflection ___________________________  Reflection ___________________________

Stretch or compression_________________  Stretch or compression_________________

Phase shift __________________________  Phase shift _________________________

Vertical shift _________________________  Vertical shift _______________________

\[ 
\begin{array}{c}
\text{Example 1:} \quad y = 2(x - 3)^3 + 1 \\
\text{Example 2:} \quad y = -\frac{1}{3}(x + 2)^3 - 4 \\
\end{array}
\]
The Rational Function

Another parent function is called the **rational function**. Its equation is \( y = \frac{1}{x} \).

Here is its graph:

![Graph of the rational function](image)

There is a vertical asymptote at \( x = 0 \) and a horizontal asymptote at \( y = 0 \).

Instead of graphing rational functions using vertical and horizontal asymptotes, we will look at the rational functions as a family of the parent function. We will use reflections, phase shifts and vertical shifts to graph the family of rational functions.

The general equation for all rational functions is:

\[
y = \frac{a}{x-h} + k
\]

where the sign of \( a \) determines a **reflection**, \( h \) determines the **phase shift** for the **vertical asymptote** and \( k \) determines the **vertical shift** for the **horizontal asymptote**. In this lesson we will not be concerned with finding exact values for the x and y intercepts. Our graph will be a rough sketch of the function.

Example 1: \( y = \frac{-1}{x} \)

\( a = -1 \) represents a reflection (the graph starts below the x-axis, starting with the positive side)

phase shift = 0

vertical shift = 0

Graph:
Example 2: \[ y = -\frac{1}{x+3} - 2 \]

Graph:

- \( a = \) 
- phase shift = 
- vertical shift = 

Example 3: Write the equation from the graph in the form \[ y = \frac{a}{x-h} + k \]

Graph:

- reflection ?
- phase shift 
- vertical shift