Practice Exam 1 – Math 2331
September, 2013

First name: _____ Last name: _____ Student ID: _____

Please write your name and student ID on the blue book. Show all the work for full credit. In this exam no calculators and no cell phones are allowed. Answer all of the following questions. The maximum score is 100.

1. (10 points) Indicate by encircling whether the following statements are true (T) or false (F). No explanation necessary.

T / F If a 3×5 coefficient matrix for a linear system has 3 pivot columns, then the linear system consistent.

T / F If a matrix has pivots in every column, the column vectors are necessarily linearly independent.

T / F A linear mapping is one-to-one if and only if the associated standard matrix has pivots in every column.

T / F If a linear transformation T maps from $\mathbb{R}^2$ to $\mathbb{R}^3$ then it is not onto.

T / F The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $[A\,\mathbf{b}]$ has a pivot position in every row.

After completing this step, detach this page and insert it in your blue book.

Each of the following problems should be answered on a separate page of your blue book.

2. (8 points) An $n \times n$ matrix A is invertible. Write 4 statements that are equivalent to this.

3. (15 points) Find the general solution of the following homogeneous system. Express your answer in parametric vector form.

$$\begin{aligned} x_1 - 3x_2 - 9x_3 + 5x_4 &= 0 \\ x_2 + 2x_3 - x_4 &= 0 \end{aligned}$$

4. (6 points) Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

is $A\mathbf{x} = \mathbf{b}$ consistent for each $\mathbf{b}$? Explain your answer.

5. (15 points) Find the augmented matrix for the system associated with the vector equation $x_1\mathbf{a_1} + x_2\mathbf{a_2} = \mathbf{b}$, where the vectors are given by

$$\mathbf{a_1} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{a_2} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix},$$

If the system is consistent, solve it and express $\mathbf{b}$ as a linear combination of $\mathbf{a_1}$ and $\mathbf{a_2}$.

6. (6 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & -2 & 1 \\ -2 & -1 & 1 \end{bmatrix}$$

which is known to be row-equivalent to

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Are the columns of A linearly independent? Explain your answer.

7. (15 points) Suppose A is a 2×2 matrix and the following row operations convert A to the 2×2 identity matrix I_2. First, rows 1 and 2 are exchanged, then row 2 is replaced by the sum of itself with (-4) times row 1. Finally, row 2 is scaled by a factor 5.

   (a) Which three elementary matrices E_1, E_2 and E_3 correspond to these three elementary row operations, respectively?

   (b) Use the elementary matrices to find A^{-1}.

   (c) Compute A.

8. (15 points) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation

$$T(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 - x_3, 3x_3 + 6x_4, x_3 - x_1 - 2x_2, x_4).$$

   (a) Find the standard matrix of T.

   (b) Is T one-to-one? Explain your answer.

9. (10 points) Suppose A is a 6×6 matrix and the equation $A\mathbf{x} = \mathbf{b}$ has no solution for some $b \in \mathbb{R}^6$. What can you say about the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Give as much information as possible.