Assignment 4, due Thursday, October 3, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1
Let $\mathbb{R}^2$ be equipped with the usual Euclidean metric. Show that the open ball $B(0, 1) = \{(x_1, x_2) : x_1^2 + x_2^2 < 1\}$ has the boundary
\[
\partial B(0, 1) = \{(x_1, x_2) : x_1^2 + x_2^2 = 1\}.
\]

Problem 2
Let $(X, d)$ be a discrete metric space. For any $A \subset X$, describe the closure, the interior and the boundary of $A$.

Problem 3
Let $X$ be a set and let $d$ and $\rho$ be two uniformly equivalent metrics on $X$. Prove that $(X, d)$ is a complete metric space if and only if $(X, \rho)$ is complete.

Problem 4
Let $\mathbb{R}$ be equipped with the usual metric and $A = \{\frac{1}{n} : n \in \mathbb{N}\}$.

a. Show that $A$ is not compact.

b. Show that $A \cup \{0\}$ is compact.