MATH 4331
Introduction to Real Analysis
Fall 2013

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 5, due Thursday, October 24, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let (X, d) be a metric space and K_1, K_2, $\ldots$, K_n be compact subsets of X. Prove that $K = K_1 \cup K_2 \cup \ldots K_n$ is compact.

Problem 2

Let (X, d) be a metric space and $K \subset X$ be compact. Prove that K is bounded.

Problem 3

Find an example for two metric spaces (X, d) and (Y, ρ), a continuous function $f : X \to Y$ and a Cauchy sequence $\{p_n\}_{n \in \mathbb{N}}$ in X which is not mapped to a Cauchy sequence in Y.

Problem 4

Let $X = [0, \infty)$ be equipped with the usual metric from $\mathbb{R}$. Show that the function $f(x) = \sqrt{x}$ is uniformly continuous on X. Hint: Use the inequality $\sqrt{a+b} \leq \sqrt{a} + \sqrt{b}$ (proved by squaring both sides).