

**MATH 4331**  
**Introduction to Real Analysis**  
**Fall 2013**

First name: \_\_\_\_\_ Last name: \_\_\_\_\_

<b>Points:</b>
----------------

## Assignment 7, due Thursday, November 7, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

### Problem 1

Prove that the only connected sets in  $\mathbb{Q}$ , when equipped with the usual metric, are the sets containing only a single point or the empty set.

### Problem 2

Let  $\mathbb{R}$  and  $\mathbb{Q}$  be equipped with the usual metric. Prove that if  $f : \mathbb{R} \rightarrow \mathbb{Q}$  is continuous, then  $f$  is constant.

### Problem 3

Prove that if a metric space  $(X, d)$  is pathwise connected, then  $(X, d)$  is connected.

### Problem 4

**Def.** Let  $g : [a, b] \rightarrow \mathbb{R}$ . By the **graph of  $g$**  we mean the set  $G = \{(x, g(x)) : a \leq x \leq b\} \subseteq \mathbb{R}^2$ .  
Let  $g : [a, b] \rightarrow \mathbb{R}$ . Prove that  $g$  is continuous if and only if the graph of  $g$  is a pathwise connected subset of the plane.