Assignment 5, due Thursday, October 1, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $f$ and $g$ be differentiable functions on an interval $(a, b)$, $a < b$. If there is $x_0$ for which $f(x_0) = g(x_0)$ and $f(x) \leq g(x)$ for all $x \in (a, b)$, prove that $f'(x_0) = g'(x_0)$.

Problem 2

Show the product rule: If $f$ and $g$ are differentiable functions on an interval $(a, b)$ and $x_0 \in (a, b)$, then $(fg)'(x_0) = f(x_0)g'(x_0) + f'(x_0)g(x_0)$.

Problem 3

If $f$ and $g$ are differentiable on $[a, b]$ and $f'(x) = g'(x)$ for all $a < x < b$, prove that $g(x) = f(x) + C$ for some constant $C \in \mathbb{R}$.

Problem 4

Assume $f$ is differentiable on $[a, b]$ and $f'(a) < 0 < f'(b)$. Show the following:

(a) There are $c, d$ with $a < c < d < b$ and $f(c) < f(a)$ as well as $f(d) < f(b)$.

(b) The minimum of $f$ on $[a, b]$ occurs at $x_0 \in (a, b)$.

(c) Hence, there is $x_0 \in (a, b)$ with $f'(x_0) = 0$.

Use this to prove that if $f$ is differentiable on $[a, b]$, and $f'(a) < L < f'(b)$, then there is $x_0 \in (a, b)$ with $f'(x_0) = L$.

Problem 5

If $f$ is differentiable on $\mathbb{R}$ and $f'$ is strictly increasing, show that $f'$ is continuous. Hint: Prove that if a function $g$ is strictly increasing, then either $\sup_{x<a} g(x) < \inf_{x>a} g(x)$ or $g$ is continuous at $a \in \mathbb{R}$.