Department of Mathematics University of Houston

MATH 4331

Introduction to Real Analysis

Fall 2015

First name: _____ **Last name:** _____ **Points:**

Assignment 9, due Thursday, December 3, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Find all intervals on which the sequence of real-valued functions $\{f_n\}_{n=1}^{\infty}$ on $\mathbb{R}$ defined by $f_n(x) = \frac{x^{2n}}{n+x^{2n}}$ converges uniformly.

Problem 2

Show that if $\sum_{n=1}^{\infty} |a_n| < \infty$, then $\sum_{n=1}^{\infty} a_n \cos nx$ converges uniformly on $\mathbb{R}$.

Problem 3

Prove that if we define for $x, y \in \mathbb{R}$, $d(x,y) = |x^2 - y^2|$ then $(\mathbb{R}, d)$ **is not** a metric space.

Problem 4

Prove that if we define for $x, y \in \mathbb{R}$, $d(x,y) = |x-y| + |x^2-y^2|$ then $(\mathbb{R}, d)$ **is** a metric space. Mention all of the properties that are required, even if some may seem obvious to you.

Problem 5

Prove that $(\mathbb{R}^2, d)$ is a metric space if we define that for any two points $(x,y), (x',y') \in \mathbb{R}^2$, their distance is

$$d((x,y),(x',y')) = \begin{cases} |y| + |y'| + |x-x'| & \text{if } x \neq x', \\ |y-y'| & \text{if } x = x'. \end{cases}$$