Department of Mathematics University of Houston

MATH 4331/6312

Introduction to Real Analysis

Fall 2017

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 2, due Thursday, September 14, 10am

Please staple this cover page to your homework. Circle your course number, 4331 or 6312. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

The interior A° of a set $A \subset \mathbb{R}^n$ is defined as largest open subset, or equivalently, as the set containing each point $x \in A$ for which there exists $r > 0$ such that $B_r(x) \subset A$.

Show that $A^\circ = (\overline{A'})'$, that is, the interior of A is obtained by taking the complement of the closure of the complement of A.

Problem 2

Suppose that A and B are subsets of $\mathbb{R}$.

a. Show that if A and B are closed, then the set $A \times B = \{(x, y) \in \mathbb{R}^2 : x \in A, y \in B\}$ is closed in $\mathbb{R}^2$.

b. Likewise, show that if A and B are both open, then $A \times B$ is open.

Problem 3

Let A and B be subsets of $\mathbb{R}^n$, $A \subset B$. The set A is dense in B if B is contained in $\overline{A}$.

a. Show that the set of irrational numbers, $\mathbb{R} \setminus \mathbb{Q}$, is dense in $\mathbb{R}$.

b. Hence, show that $\mathbb{Q}$ has empty interior.

Problem 4

Show that the union of finitely many compact sets C_1, C_2, ..., C_m in $\mathbb{R}^n$ is compact.

Problem 5

Show that the intersection of any family of compact sets $\{C_i\}_{i \in I}$ in $\mathbb{R}^n$ is compact.