Practice Exam 1 – Math 4331/6312
October, 2019

First name: ________________ Last name: ________________ Last 4 digits of Student ID: _______

1 True-False Problems

Put a T in the box beside each statement that is true, an F if the statement is false.

- ☐ Every convergent sequence in $\mathbb{R}^n$ is bounded.
- ☐ Every convergent sequence in $\mathbb{R}^n$ is Cauchy.
- ☐ If U is an open subset and C is a closed subset of $\mathbb{R}^n$, then $U \setminus C$ is an open subset of $\mathbb{R}^n$.
- ☐ The Cantor set is compact.
- ☐ If $S = [0,1) \cup (1,2]$, then S is a connected subset of $\mathbb{R}$.
- ☐ If $f : S \subset \mathbb{R}^m \to \mathbb{R}^n$ is continuous, then $f^{-1}(U)$ is open in $\mathbb{R}^m$ for any set U that is open in $\mathbb{R}^n$.
- ☐ If $K \subseteq \mathbb{R}^n$ has the property that every convergent sequence in K is bounded, then K is compact.
- ☐ If $S = \{(x,y) : 0 \le y \le \frac{1}{x}, x > 0\} \cup \{(0,y) : y \in \mathbb{R}\}$ then S is a closed set.

After completing this part, hand it in to obtain the remaining portion of the exam.

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In the following problems, you may quote results from class to simplify your answers. You do not need to include a proof of a statement if it was discussed in class.

2 Problem

Prove that if a subset A of $\mathbb{R}^n$ has the property that every Cauchy sequence $\{a_k\}_{k=1}^{\infty}$ in A converges and has a limit $a \in A$, then A is a closed set.

3 Problem

Let $p_k = (\frac{1}{k}, \frac{1}{k^2})$, $k \in \mathbb{N}$, define a sequence in $\mathbb{R}^2$. Prove that $\lim_{k\to\infty} p_k = (0, 0)$.

4 Problem

Let $f : [a, b] \to \mathbb{R}$, $a < b$, be a continuous function. Recall that the graph of f is the set

$$G = \{(x, f(x)) : a \leq x \leq b\} \subset \mathbb{R}^2 .$$

Show that G is a closed set in $\mathbb{R}^2$.

5 Problem

Let $K \subseteq \mathbb{R}$ be a compact set. Explain why there is $x \in K$ with $x \geq y$ for any $y \in K$.

6 Problem

(a) Consider a function $f : S \subset \mathbb{R}^m \to T \subset \mathbb{R}^n$. State the definition of Lipschitz continuity for f.

(b) Prove that if $f : S \subset \mathbb{R}^m \to T \subset \mathbb{R}^n$ and $g : T \subset \mathbb{R}^n \to \mathbb{R}$ are both Lipschitz continuous on their domains, then the composition $h = g \circ f, h(x) = g(f(x))$ is Lipschitz continuous on S.

(Problem 6, continued)

7 Problem

Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Show that the graph $G = \{(x, f(x)) : x \in \mathbb{R}\}$ has a complement $X = \mathbb{R}^2 \setminus G$ that is disconnected.

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