Assignment 8, due Thursday, April 21, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Compute the Fourier series of the following functions on \([-\pi, \pi]\):

a. \(f(t) = \cos^3(t)\),

b. \(f(t) = |\sin t|\),

c. \(f(t) = t\).

Problem 2

If \(f\) is a \(2\pi\)-periodic function with known Fourier series, let \(\alpha \in \mathbb{R}\) and define \(g\) by \(g(t) = f(t - \alpha), t \in \mathbb{R}\). Express the Fourier series of \(g\) in terms of that of \(f\). Use this together with Problem 1.b to find the Fourier series of \(g(t) = |\cos(t)|\).

Problem 3

Show that if \(f\) is continuous and \(2\pi\)-periodic with Fourier coefficients such that \(\sum_{k=1}^{\infty} (|a_k| + |b_k|) < \infty\), then the Fourier series of \(f\) converges uniformly to it.

Problem 4

Show that the Fourier series for \(f(x) = x^2\) on \([-\pi, \pi]\) is given by

\[
\frac{\pi^2}{3} - 4 \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\cos(kx)}{k^2}
\]

and using the result on (uniform) convergence together with an appropriate choice of \(x\), show

\[
\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}.
\]