 Assignment 2, due Thursday, February 8, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $\mathbb{R}$ be equipped with the usual metric and $A = \{\frac{1}{n} : n \in \mathbb{N}\}$.

a. Show that $A$ is not compact.

b. Show that $A \cup \{0\}$ is compact without appealing to the characterization of compactness in Euclidean spaces.

Problem 2

Let $(X, d)$ be a metric space and $K_1, K_2, \ldots, K_n$ be compact subsets of $X$. Prove that $K = K_1 \cup K_2 \cup \ldots K_n$ is compact.

Problem 3

Let $(X, d)$ be a metric space and $K \subset X$ be compact. Prove that $K$ is bounded.

Problem 4

Let $(X, d)$ be a metric space, $Y \subset X$ and consider the metric space $(Y, d)$.

a. Show that every open set $U$ in $Y$ has the form $U = V \cap Y$ for an open set $V$ in $X$. Hint: Show this first for open balls.

b. Show that $Y$ is compact in $(Y, d)$ if and only if every collection of open sets $\{V_j\}_{j \in J}$ in $X$ that covers $Y$ has a finite subcover.