Assignment 3, due Thursday, February 15, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Find an example for two metric spaces \((X, d)\) and \((Y, \rho)\), a continuous function \(f : X \to Y\) and a Cauchy sequence \(\{p_n\}_{n \in \mathbb{N}}\) in \(X\) which is not mapped to a Cauchy sequence in \(Y\).

Problem 2

Let \((X, d)\) and \((Y, \rho)\) be metric spaces. Prove that if \(f : X \to Y\) is continuous, then for any set \(A\) in \(X\) with closure \(\overline{A}\),

a. we have \(f(\overline{A}) \subseteq \overline{f(A)}\)

b. and this inclusion can be proper, i.e. give an example for which \(f(\overline{A}) \neq \overline{f(A)}\).

Problem 3

Let \((X, d)\) and \((Y, \rho)\) be metric spaces, \(X\) compact. Show that if \(f : X \to Y\) is continuous, one-to-one and onto, then \(f^{-1}\) is continuous.

Problem 4

Show that the completion of a metric space \((X, \rho)\) is compact if and only if \(X\) is totally bounded.