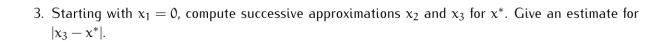
Practice Exam 2 - Math 4332/6313 March, 2018

First name:	Last name:	Last 4 digits student ID:
	roblems, you may quote stater proof of a statement if it was	ments from class to simplify your answers. Yo discussed in class.
1 Problem		
Let $f(x) = \frac{1}{2}e^{-x} - \frac{1}{4}$.		
1. Briefly explain v	vhy the function f has a single	root in the interval [0, 1].
2. Set up Newton's [0, 1].	s method to find the root x^* of	f. Show that Newton's map is a contraction of



Consider the ODE $f'(x) = 1 + \frac{1}{2}f(x)$ with initial value f(0) = 1 on [0, 1].

1. Use an integral to define the map $T:C([0,1])\to C([0,1])$ as in class.

2. Show that T is a contraction on $C([0,1])$, equipped with the (usual) max-metric.												

3. Starting with $f_1(x)=1$, compute $f_3=T^2f_1$. If f^* is the unique solution to the ODE, estimate the distance $\|f_3-f^*\|_{\infty}$.

Show that if (X,d) is a compact metric space and $f:X\to X$ satisfies d(f(x),f(y))< d(x,y) for all $x,y\in X$ then f has a fixed point.

Find the Taylor polynomial of order 2 of the function $f(x) = \sin(x)$, $\alpha = \pi/2$. Estimate the error when $f(3\pi/5)$ is replaced by $P_2(3\pi/5)$.

Show that if f is an even function in C([-1,1]), then for any $\varepsilon>0$ there is an even polynomial p, that is, p(x)=p(-x) for $x\in [-1,1]$, such that $\|f-p\|_{\infty}<\varepsilon$. Hint: If q is any polynomial, then $p(x)=\frac{1}{2}(q(x)+q(-x))$ is an even polynomial.

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