Assignment 4, due Thursday, February 13, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1
Let \((X, d)\) be a metric space and \(K \subset X\) be compact. Prove that \(K\) is bounded.

Problem 2
Let \(\mathbb{R}\) be equipped with the usual metric and \(A = \{\frac{1}{n} : n \in \mathbb{N}\}\).

a. Show that \(A\) is not compact.

b. Show that \(A \cup \{0\}\) is compact.

Problem 3
Let \((X, d)\) be a metric space and \(K_1, K_2, \ldots, K_n\) be compact subsets of \(X\). Prove that \(K = K_1 \cup K_2 \cup \ldots K_n\) is compact.

Problem 4
Let \((K, d)\) be a compact metric space and let \(C_0 = K, C_j \supset C_{j+1}\) for each \(j \in \mathbb{N}\) define a nested sequence of closed, non-empty sets, then show \(\bigcap_j C_j \neq \emptyset\). Hint: Use the finite subcover property of \(K\) in an indirect proof with \(U_j = C_j\).