1. F ( discrete metric)
T ( compact )
T ( in discrete space, any set open )
F ( no content )
T ( closedness under finite unions )
F $\overline{B}_1(0)$ in $C(K)$
2. By $K_1, K_2$ compact, they are closed.

Thus, $K = K_1 \cap K_2$ is closed.

Now $K \subset K_1$, so $K$ is closed subset of compact set, hence compact.

3. We have by $L$-Lipschitz with $L \leq 1$,
for $x \in [0,1]$, $f \in K$,

$$|f(x)| \leq L\,|x-0| \leq |x-0| \leq |x|$$

so $f \in \overline{B}_1(0)$, hence $K$ bounded.

Also, $K$ is closed, because if $(f_n)_n$ is a sequence of $L$-Lipschitz cont.

has $f_n \to f$, then for $x,y \in [0,1]$, by uniform conv.

$$|f(x) - f(y)| = \lim_{n \to \infty} |f_n(x) - f_n(y)| \leq |x-y|$$

$$\leq |x-y| \Rightarrow f \text{ is } L \text{-Lipschitz}$$
moreover, \( f_n(0) \to f(0) = 0 \)

so \( f \in K \).

Finally, \( K \) is an equicontinuous family, because for any \( x \in [0,1] \), given \( \varepsilon > 0 \), letting \( \delta = \varepsilon \) gives that for \( y \in [0,1] \) with \( |y - x| < \delta \), for any \( f \in K \),

\[
|f(y) - f(x)| \leq |x - y| < \delta = \varepsilon.
\]

Thus, \( K \) is clearly bounded and consists of equicontinuous family, so \( K \) is compact.
4. If $A$ is totally bounded, then for any $\varepsilon > 0$, there is finite set with $\bigcap_{n=1}^{N} B_{\varepsilon/2}(x_n) \subseteq A$.

We also know that if $y \in \overline{A}$, then for any $\varepsilon > 0$, $A \cap B_{\varepsilon/2}(y) \neq \emptyset$.

Thus, there is $x \in A$ with $d(x, y) < \frac{\varepsilon}{2}$.

We conclude, then there is $x_i$, i.e., $1, 2, \ldots, N$ such that $d(x_i, x) < \frac{\varepsilon}{2}$, and hence

$$d(x_i, y) \leq d(x_i, x) + d(x, y) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

Thus, $\bigcap_{n=1}^{N} B_{\varepsilon}(x_n)$, and so $\overline{A}$ is totally bounded.
5. If \( U \) is open w.r.t. usual metric, then \( f^{-1}(U) = U \) is open in discrete metric, so \( f \) is cont. Also, \( f(x) = f(y) \) implies \( x = y \), so \( f \) is 1-1, but \( f^{-1} \) is not continuous because \( \emptyset \) is open (w.r.t. discrete metric), but \( (f^{-1})^{-1}(\emptyset) = \{ f(0) \} = \{ 0 \} \) is not open w.r.t. usual metric.

6. By continuity and compactness of \( K \),
\[
\sup_{x \in K} |f_i(x)| < M_i < \infty \quad \text{so for } i = 1, 2.
\]
Also, it is enough to show continuity of \( f = f_1 f_2 \), since domain compact.

For \( \varepsilon > 0 \), we know there are \( \delta_1, \delta_2 > 0 \) s.t. for all \( y \in K \) with \( d(y, x) < \delta_i \),
\[
|f_i(y) - f_i(x)| < \frac{\varepsilon}{\text{Hit}_i(i, 1)} \quad i \in \{1, 2\}.
\]
Then, if $d(y, x) < \delta \equiv \min \{\delta_1, \delta_2\}$

$$|f_1(y)f_2(y) - f_1(x)f_2(x)|$$

$$= |f_1(y)f_2(y) - f_1(y)f_2(x) + f_1(y)f_2(x) - f_1(x)f_2(x)|$$

$$\leq |f_1(y)||f_2(y) - f_2(x)| + |f_2(x)||f_1(y) - f_1(x)|$$

$$\leq M_1 |f_2(y) - f_2(x)| + M_2 |f_1(y) - f_1(x)|$$

$$< M_1 \frac{\varepsilon}{M_1 + M_2 + 1} + M_2 \frac{\varepsilon}{M_1 + M_2 + 1}$$

$$< \varepsilon$$

so $f = f_1f_2$ is continuous at $x$. This implies $f$ is uniformly continuous on $U$, by compactness.

7. Given Cauchy seq. in $(Y, d)$, then it is Cauchy in $(\mathbb{R}, d)$ and has a limit. Since $Y \subset [0, 1]$ is closed, the limit is included in $Y$, so Cauchy seq. converges in $Y$. Thus, $(Y, d)$ is complete.