1 In each of the following cases, $T$ is the linear operator on $\mathbb{R}^2$ which has the matrix representation $[T]_E = A$ with respect to the standard ordered basis $E$ for $\mathbb{R}^2$. Moreover, $U$ is the linear operator on $\mathbb{C}^2$ with the same matrix representation, $[U]_E = A$, with respect to the standard ordered basis $E$ for $\mathbb{C}^2$.

(a) Given

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

find the eigenvalues for $T$ and $U$. For each eigenvalue $\lambda$, find a basis for the corresponding eigenspace $\ker(T - \lambda \text{id})$ in $\mathbb{R}^2$ and $\ker(U - \lambda \text{id})$ in $\mathbb{C}^2$.

(b) Find all the eigenvalues and for each eigenvalue a basis for the corresponding eigenspace of $T$ and $U$ if they have the matrix representation

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}.$$ 

It is OK to be a bit surprised.

4 Let $T$ be the linear operator on $\mathbb{R}^3$ which is represented by the matrix

$$[T]_E = \begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$$

with respect to the standard ordered basis $E$. Prove that $T$ is diagonalizable by finding a basis $B = \{\beta_1, \beta_2, \beta_3\}$ for $\mathbb{R}^3$ which consists of eigenvectors for $T$.

10 Prove that $T$ on $\mathbb{R}^2$ is diagonalizable if it has the matrix representation

$$[T]_E = \begin{pmatrix} a & b \\ b & c \end{pmatrix},$$

with respect to the standard ordered basis $E$ for some $a, b, c \in \mathbb{R}$. 