First Name: ____________________

Last Name: ____________________

Signature: ____________________

Student I.D. No.: ____________________

Math 6321 Practice Midterm Exam
In Class Part

March, 2011
One hour and twenty minutes

University of Houston

Instructions:

1. Put your name, signature and I.D. No. in the blanks above.
2. There are **three questions** in this in-class part of the exam. Answer the questions in the spaces provided, using the backs of pages or the blank pages at the end for overflow or rough work.
3. Your grade will be influenced by how clearly you present your solutions. **Justify your solutions carefully** by referring to definitions and results from class where appropriate.
4. **This is a closed book exam.**

1. (a) State Rudin's version of the Banach-Steinhaus theorem.

   (b) State the polar decomposition theorem.

2. Let X be a normed linear space. Show that X is complete if and only if whenever $\sum_{j=1}^{\infty} \|x_n\| < \infty$, then $\sum_{j=1}^{\infty} x_j$ converges to some $s \in X$.

3. Let Λ be a continuous linear functional on $L^1([-1,1])$, the space of Lebesgue-integrable functions on $[-1,1]$, and assume $\Lambda(g) = 0$ if g is odd, so $g(-x) = -g(x)$ for almost every $x \in [0,1]$. Show that there exists $h \in L^\infty([-1,1])$ with

$$\Lambda(f) = \int_{[-1,1]} f(x)h(x)dx$$

and $h(-x) = h(x)$ almost everywhere. If you quote theorems from class, explain that the relevant assumptions are satisfied. Hint: every function on $[-1,1]$ can be written as a sum of an even and an odd function.

First Name: ______________________

Last Name: ______________________

Signature: ______________________

Student I.D. No.: ______________________

Math 6321 Practice Midterm Exam
Take-home Part

March, 2011

due on Thursday, 2:30pm

University of Houston

<u>Instructions:</u>

1. Put your name, signature and I.D. No. in the blanks above.
2. There are **two questions** in this take-home part of the exam. Answer the questions in the spaces provided, using the backs of pages or the blank pages at the end for overflow or rough work.
3. Your grade will be influenced by how clearly you present your solutions. **Justify your solutions carefully** by referring to definitions and results from class where appropriate.
4. **This is a closed book exam. You are not permitted to communicate with anyone except the instructor (bgb@math.uh.edu) about questions on this part of the exam, or about any other material related to these questions. Your signature on this page indicates your agreement to these conditions of the exam.**

1. Prove the following without referring to the Radon-Nikodym theorem: If λ and μ are finite positive measures on a measurable space $(X, \mathcal{M})$ and λ is absolutely continuous with respect to μ, then either $\lambda \equiv 0$ or there exists $\epsilon > 0$ and $E \in \mathcal{M}$ with $\mu(E) > 0$ such that for all $A \in \mathcal{M}$,

$$\lambda(A) \geq \epsilon \int_A \chi_E d\mu \,.$$

2. Assume f is a bounded, real-valued Lebesgue-measurable function on $[0,1]$ such that

$$\int_{[0,1]} x^n f(x)dx = 0$$

for all $n \in \{0,1,2,3,\ldots\}$. Prove that $f(x) = 0$ for Lebesgue-almost every $x \in [0,1]$.