MATH 6360

Applied Analysis

Fall 2018

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 9, due Friday, November 30, 10am

Please staple this problem sheet to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Explain why $C([0,1])$, equipped with the L^p-norm for $1 \leq p < \infty$ and $p \neq 2$ cannot become an inner product space for which the norm is induced by the inner product.

Problem 2

Let H be a Hilbert space and F a bounded linear functional on H. Show that there exists a unique $z \in H$ such that $Fx = \langle x, z \rangle$ and $\|F\| = \|z\|$. Hint: The kernel of F is a closed subspace. If $F \neq 0$, consider $z' = x - Px$ where P is the projection onto the kernel of F and x satisfies $Fx \neq 0$.

Problem 3

Find

$$\min\left\{\int_{-1}^{1} |x^2 - a - bx|^2 dx : a, b \in \mathbb{R}\right\}.$$

Problem 4

Find the function $f \in C([-\pi, \pi])$, which satisfies

$$\int_{-\pi}^{\pi} f(x) x dx = 1 \text{ and } \int_{-\pi}^{\pi} f(x)\sin(x) dx = 2$$

and has minimal L^2-norm. Show that this function is unique.