Department of Mathematics University of Houston

MATH 6361
Applied Analysis
Spring 2019

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 2, due Friday, February 1, 10am

Please staple this problem sheet to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Show that the Fourier series for the function $f(x) = x^2$ on $[-\pi, \pi]$ is given by

$$f(x) = \frac{\pi^2}{3} + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}(e^{inx} + e^{-inx})$$

which holds pointwise for each $x \in [-\pi, \pi]$. Choose a suitable x to show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6$.

Problem 2

Is every complete orthonormal system in a Hilbert space a Schauder basis? Explain the reasons for your answer.

Problem 3

Show that for $j, k \in \mathbb{Z}$,

$$\phi_0(x) = 1$$

and

$$\psi_{j,k}(x) = 2^{j/2}\psi_{0,0}(2^j x - k)$$

with $\psi_{0,0}(x) = 1$, if $0 \le x < 1/2$ and $\psi_{0,0}(x) = -1$ if $1/2 \le x \le 1$ defines an orthonormal system $\{\phi_0, \psi_{j,k} : 0 \le k \le 2^j - 1\}$. Recall that each element in $C([0,1])$ is uniformly continuous and use this to show that the system is actually an orthonormal basis.

Problem 4

Let H be a Hilbert space. Show that the sequence $(x_n)_{n=1}^{\infty}$ converges weakly to x_0 in H if and only if for each fixed $k \in \mathbb{N}$, $\langle x_n, x_k\rangle \to \langle x_0, x_k\rangle$. Hint: Let $Y = \{y \in H : \langle x_n, y\rangle \to \langle x_0, y\rangle\}$ then this is a subspace. Why is Y closed? Apply the orthogonal projection onto Y to convert between weak convergence in H and weak convergence in Y.