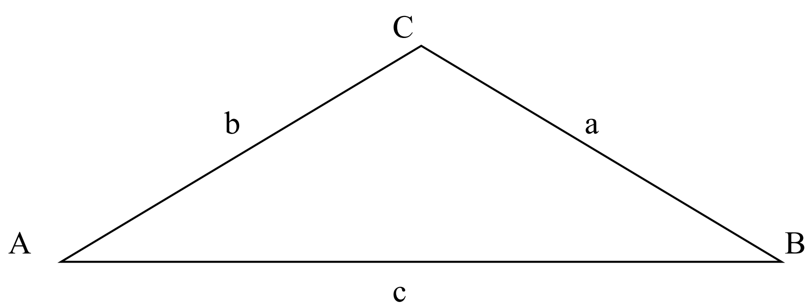
Math.1330 – Section 7.2
Area of a Triangle

In this section, we'll use a familiar formula and a new formula to find the area of a triangle. You have probably used the formula

$$Area = \frac{1}{2}\ (base)\ (height) = \frac{1}{2}\ bh$$

to find the area of a triangle, where *b* is the length of the base of the triangle and *h* is the height of the triangle. We will use this formula in some of the examples here, but we may have to find either the base or the height using trig functions before proceeding.

Here's another approach to finding area of a triangle. Consider this triangle:



The area of the triangle ABC is: $Area = \frac{1}{2}\ \mathrm{bc} \sin A$

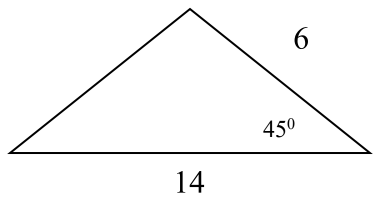
It is helpful to see this as $\boldsymbol{Area = \frac{1}{2} * side * side * sine\ of\ angle\ included.}$

Other versions that depend on the choice of the angle are:

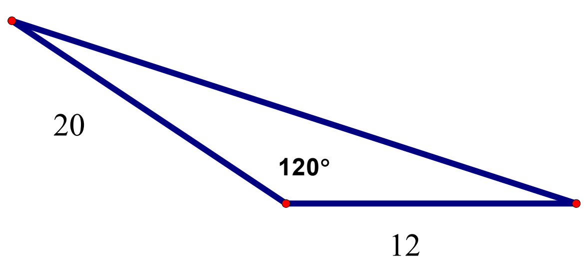
$Area = \frac{1}{2}\ ab \sin C$

$Area = \frac{1}{2}\ ac \sin B$

Example 1: Find the area of the triangle.

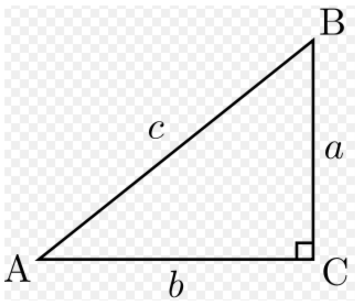


Example 2: Find the area of the triangle.



Special Cases

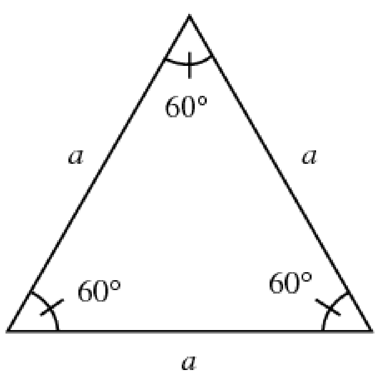
Right Triangles' Areas



$$Area = \frac{1}{2} \cdot a \cdot b \cdot \sin(90°) = \frac{1}{2} \cdot a \cdot b$$

$$Area = \frac{1}{2} \cdot leg1 \cdot leg2$$

Equilateral Triangles' Areas



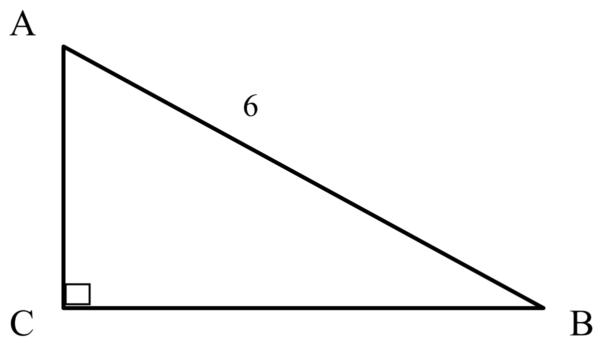
$$Area = \frac{1}{2} \cdot a \cdot a \cdot \sin(60°) = \frac{1}{2} \cdot a^2 \cdot \frac{\sqrt{3}}{2}$$

$$Area = \frac{\sqrt{3}}{4} \cdot a^2$$

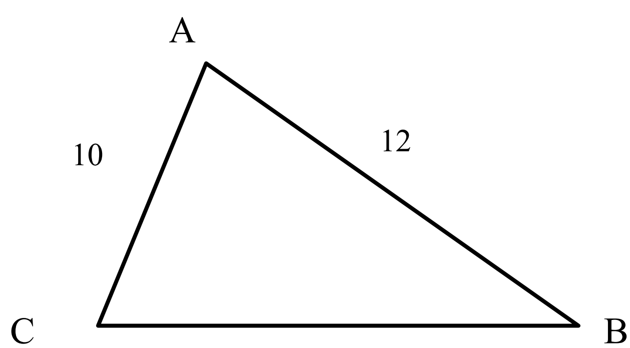
Example 3: Find the area of an isosceles triangle with legs measuring 12in. and base angles measuring $52°$ each. Round to the nearest hundredth.

Example 4: In triangle ABC; $a = 12, b = 20$ and $\sin(C) = 0.42$. Find the area of the triangle.

Example 5: In the right triangle ABC, AB=6 and $\sin(B) = \frac{1}{3}$.
Find the area of this triangle.



Example 6: In the triangle ABC, $AC = 10$, $BC = 12$, x is an acute angle such that $\sin(x) = \frac{1}{4}$. Given that $m\angle A = 2x$, find the area of this triangle.



Example 7. In triangle KLM, $k = 10$ and $m = 8$. Find all possible measures of the angle L

a) if the area of the triangle is 20 unit squares.

b) if the area of the triangle is 25 unit squares.

c) if the area of the triangle is 40 unit squares.

d) if the area of the triangle is 80 unit squares.

Formula for Area of a Regular Polygon Given a Side Length

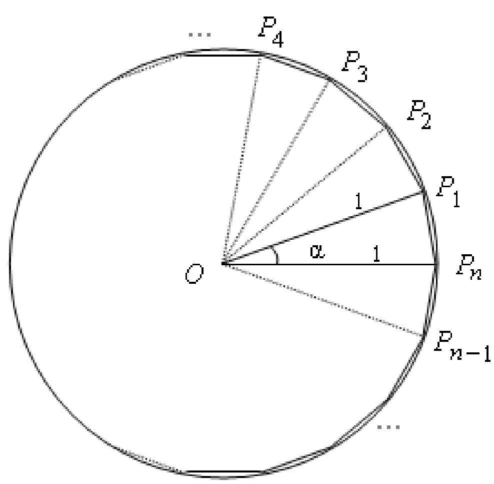
A **regular polygon** is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length). Regular polygons may be **convex** or **star**.

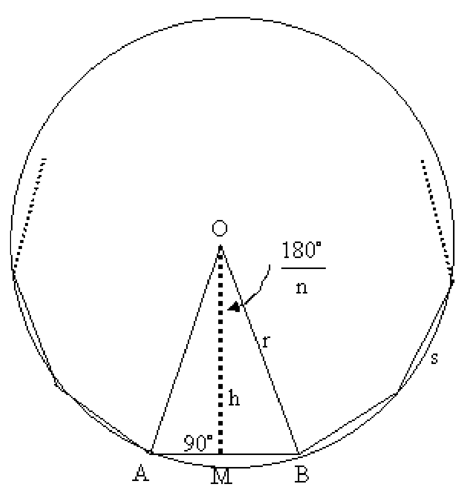
For reference, a pentagon has 5 sides, a hexagon has 6 sides, a heptagon has 7 sides, an octagon has 8 sides, a nonagon has 9 sides and a decagon has 10 sides.

The area of a regular polygon with N sides, given the length of a side S is:

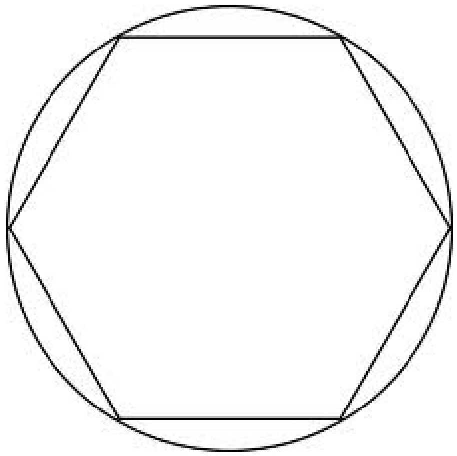
$$A = \frac{S^2 N}{4\tan\left(\frac{\pi}{N}\right)} \quad .$$

How to get this formula:



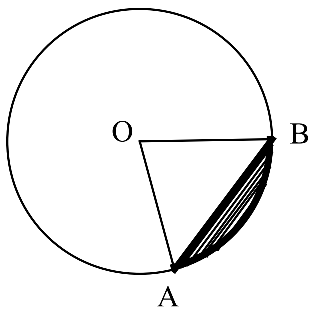


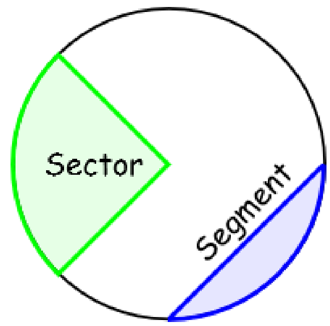
Example 8: A regular hexagon is inscribed in a circle of radius 12. Find its area.



Area of a segment of a circle

You can also find **the area of a segment of a circle.** The shaded area of the picture is an example of a segment of a circle.





To find the area of a segment with central angle θ, first we find the area of the sector it corresponds, and then we subtract the area of the triangle with that same central angle.

Area of segment = Area of sector AOB - Area of ΔAOB

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta)$$

Example 8: Find the area of the segment of the circle with radius 8 inches and central angle measuring $\frac{\pi}{4}$.