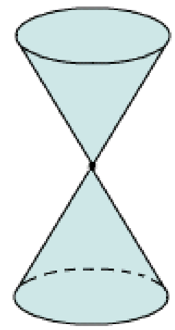
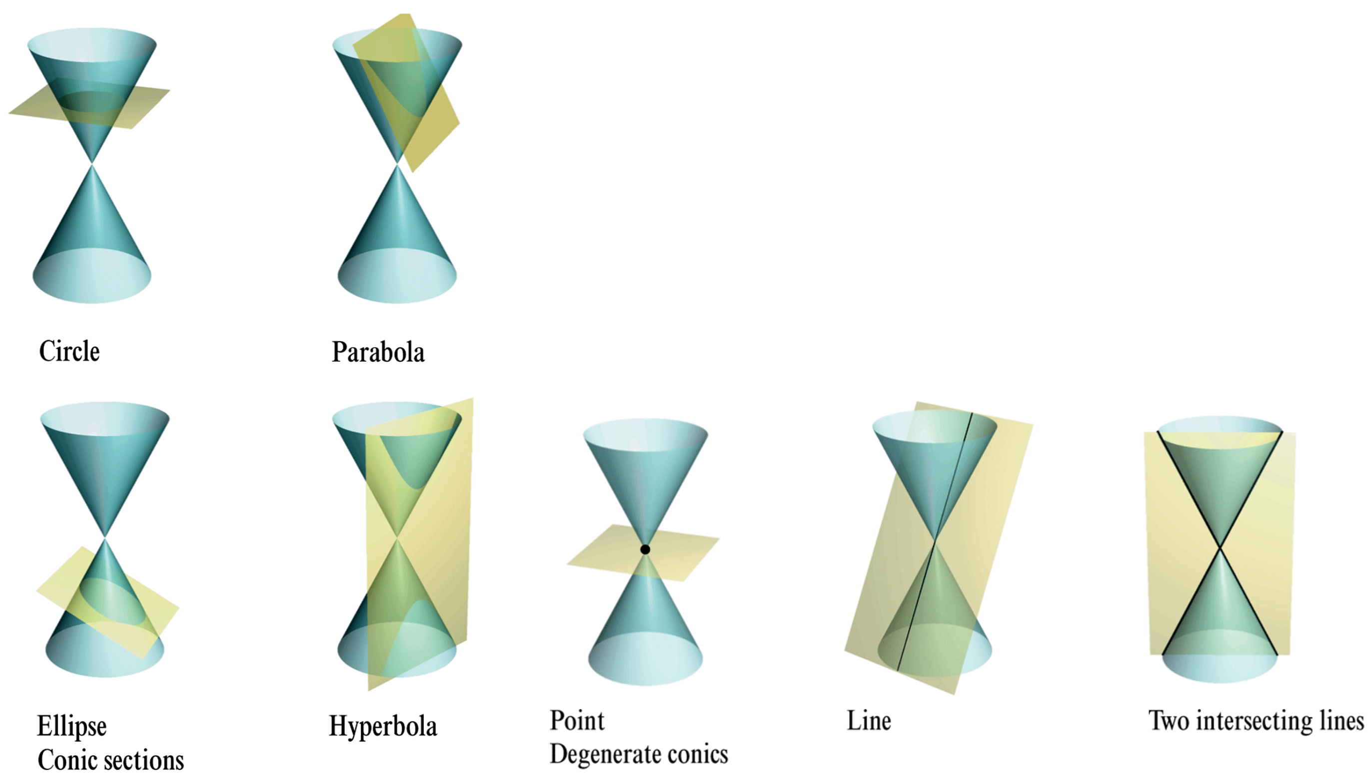
Math 1330 – Chapter 8
Introduction to Conic Sections

In this chapter, we will study conic sections (or conics). It is helpful to know exactly what a conic section is. We start by looking at a double cone. Think of this as two "pointy" ice cream cones that are connected at the small tips:



To form a conic section, we'll take this double cone and slice it with a plane. When we do this, we'll get one of several different results.



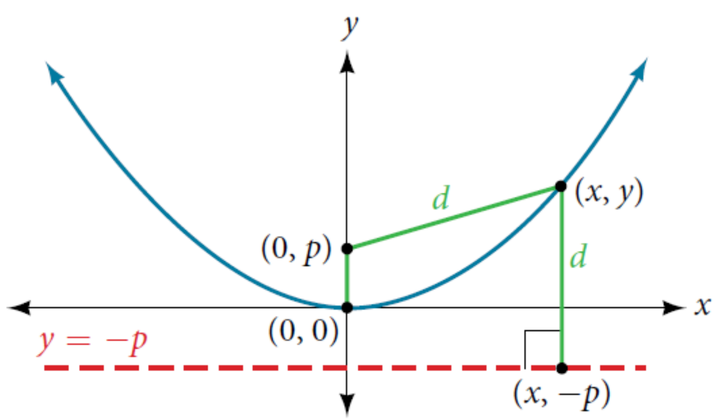
As we study conic sections, we will be looking at special cases of the general second-degree equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

Math 1330 – Section 8.1
Parabolas

Next, we'll look at parabolas. We previously studied parabolas as the graphs of quadratic functions. Now we will look at them as conic sections. There are a few differences. For example, when we studied quadratic functions, we saw that the graphs of the functions could open up or down. As we look at conic sections, we'll see that the graphs of these second degree equations can also open left or right. So, not every parabola we'll look at in this section will be a function.

We already know that the graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola. But there is more to be learned about parabolas.

Definition: A *parabola* is the set of all points equally distant from a fixed line and a fixed point not on the line. The fixed line is called the *directrix*. The fixed point is called the *focus*.



The *axis of the parabola* runs through the focus and is perpendicular to the *directrix*. The *vertex* is the point **halfway between** the *focus* and the *directrix.*

We will be working just with "horizontal" and "vertical" parabolas.

Basic "Vertical" Parabola:

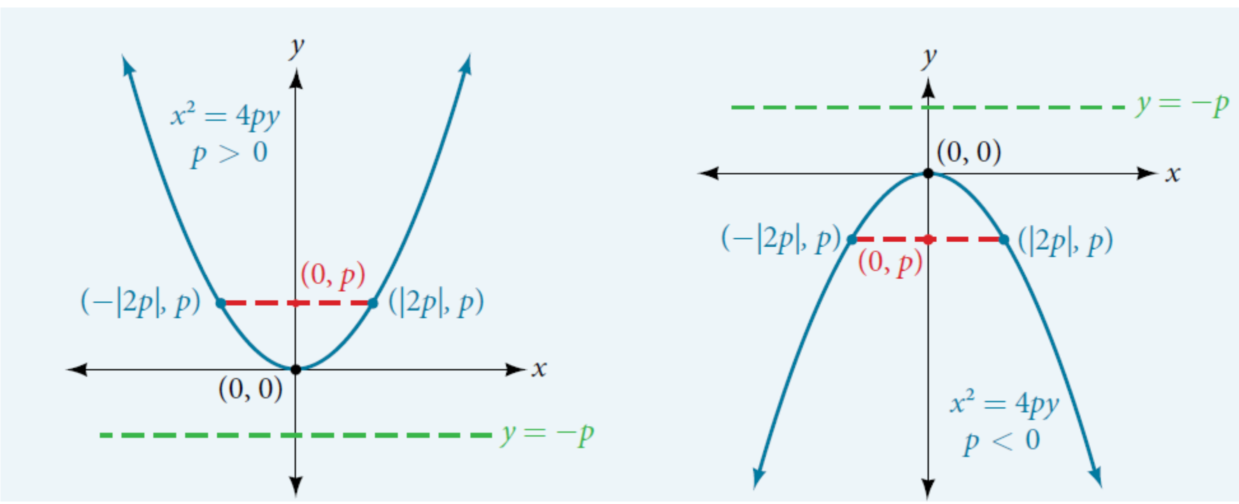
Equation: $x^2 = 4py$

Focus: $(0, p)$

Directrix: $y = -p$

Focal Width: $|4p|$

The line segment that passes through the focus and it is perpendicular to the axis with endpoints on the parabola, is called the focal chord, and the focal width is the length of the focal chord.



Note: *Vertical parabolas are quadratic functions defined by* $y = f(x) = \dfrac{x^2}{4p}$.

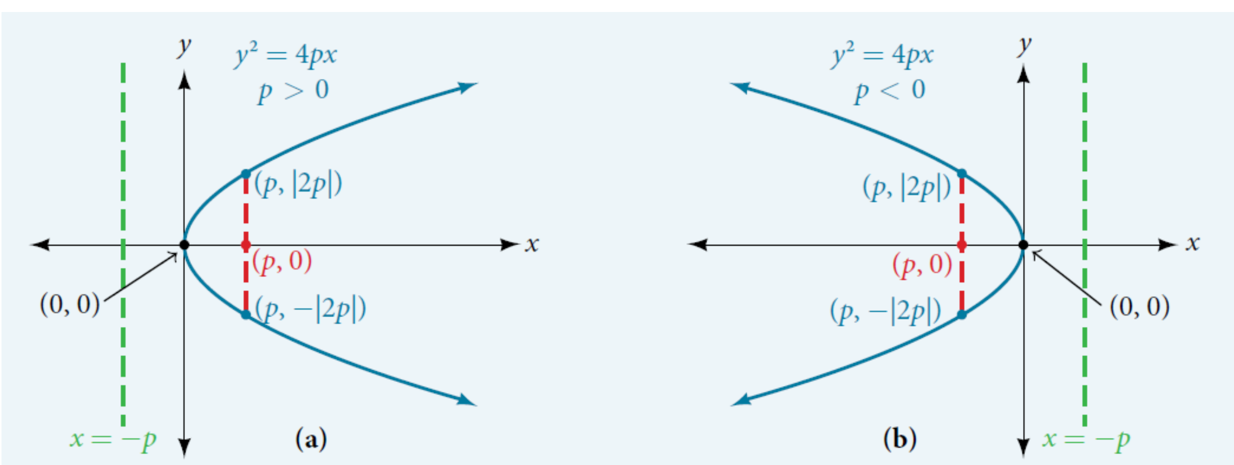
Basic "Horizontal" Parabola:

Equation: $y^2 = 4px$

Focus: $(p, 0)$

Directrix: $x = -p$

Focal Width: $|4p|$



Note: *This is not a function (fails vertical line test). However, the top half* $y = \sqrt{x}$ *is a function and the bottom half* $y = -\sqrt{x}$ *is also a function.*

Graphing parabolas with vertex at the origin:

- If the equation has x^2, it's a vertical parabola.
  If it has y^2, it's a horizontal parabola.

- Bring the equation in its standard form, i.e. $y^2 = 4px$ or $x^2 = 4py$.

- Find p.

- Determine the direction it opens.
  - If p is positive, it opens right (for $y^2 = 4px$) or up (for $x^2 = 4py$).
  - If p is negative, it opens left (for $y^2 = 4px$) or down (for $x^2 = 4py$).

- Starting at the origin, place the focus p units to the inside of the parabola over the axis of parabola.

- Place the directrix p units to the outside of the parabola, i.e. draw $y = -p$.

- Use p to find the endpoints of the focal chord, which determines the correct width of parabola. The focal width is $4p$ ($2p$ on each side of the axis) .

Example 1: Write $y^2 - 20x = 0$ in standard form and graph it.

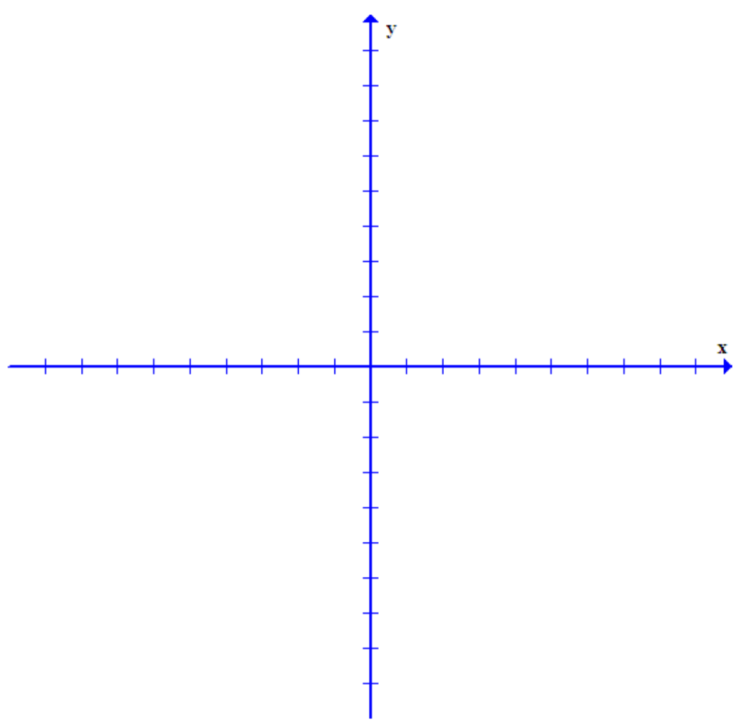
Vertex:

Focus:

Directrix:

Focal width:

Endpoints of focal chord:



Example 2: Write $6x^2 + 24y = 0$ in standard form and graph it.

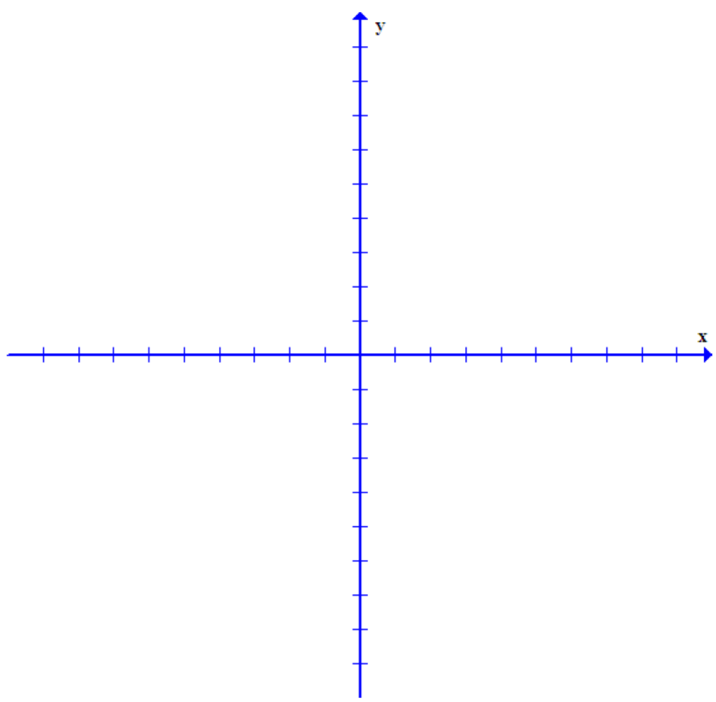
Vertex:

Focus:

Directrix:

Focal width:

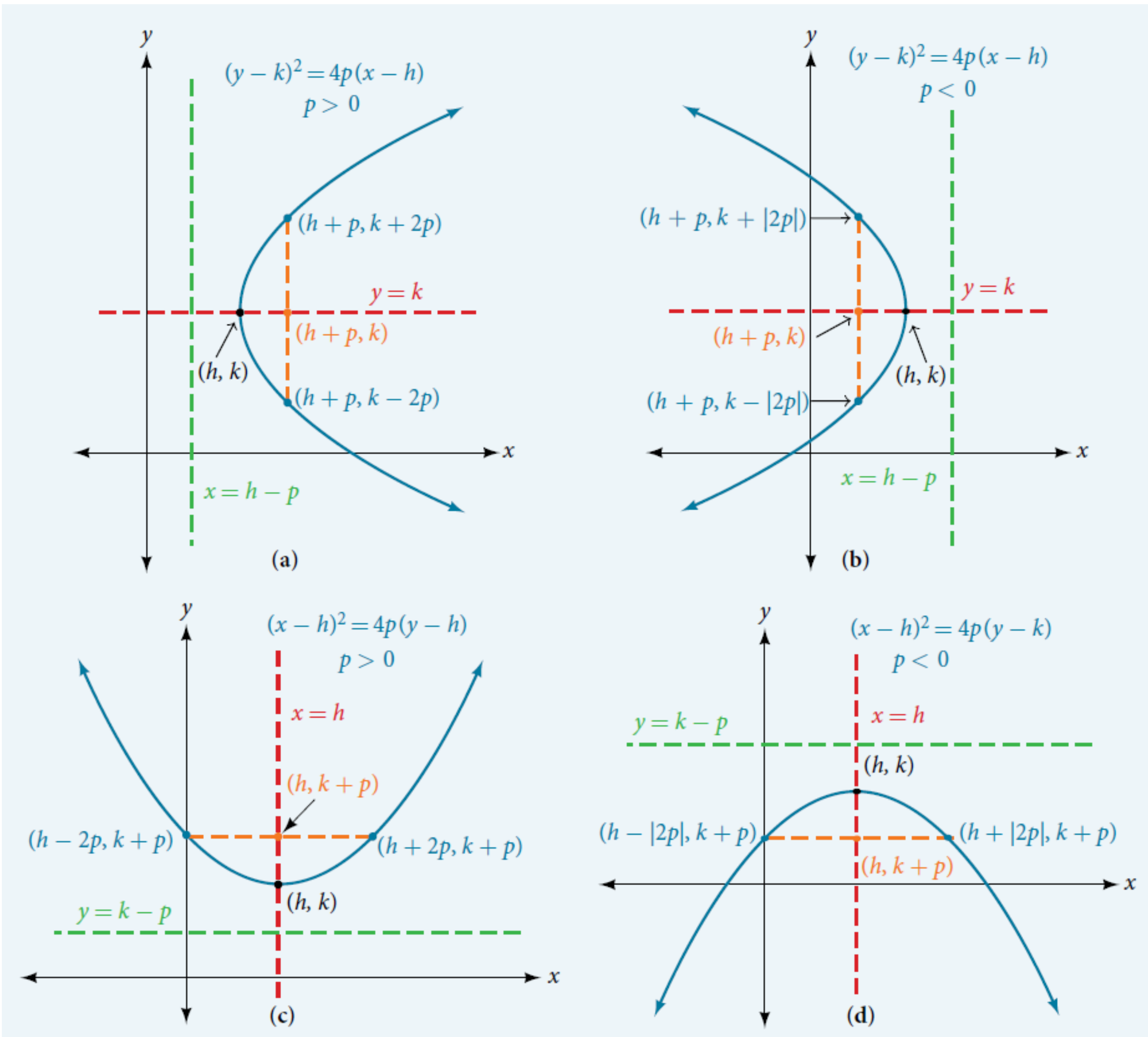
Endpoints of focal chord:



Graphing parabolas with vertex not at the origin:

- Rearrange (complete the square) to look like $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$.

- Vertex is (h,k). Draw it the same way, except start at this vertex.



Example 3: Write $y^2 - 6y = 8x + 7$ in standard form and graph it.

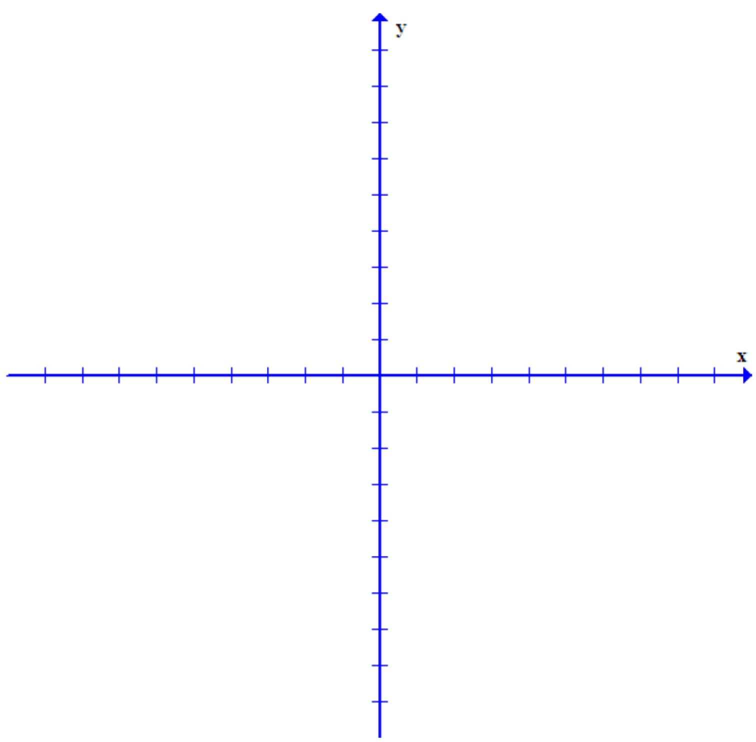
Vertex:

Focus:

Directrix:

Focal width:

Endpoints of focal chord:



Example 4: Write $x^2 + 10x = 4y - 1$ in standard form and graph it.

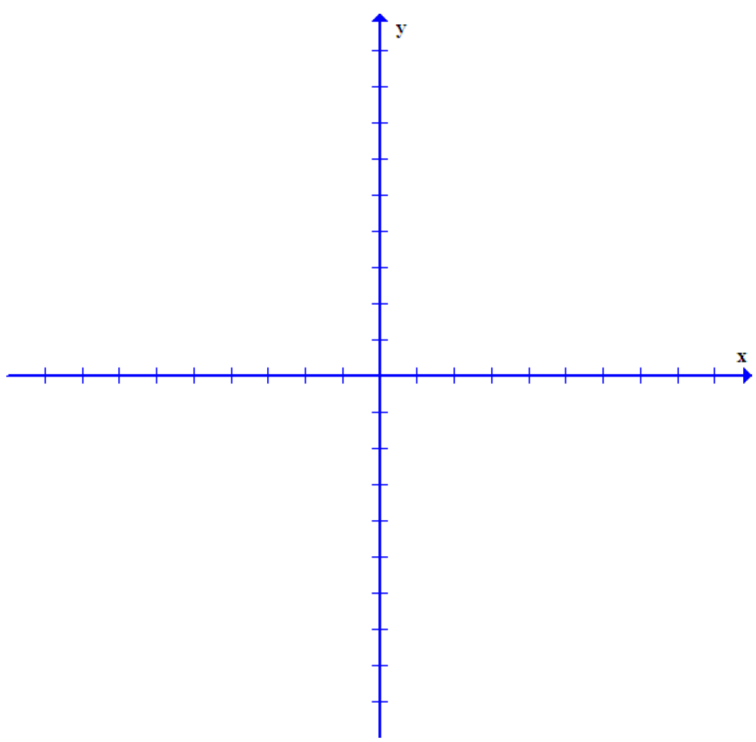
Vertex:

Focus:

Directrix:

Focal width:

Endpoints of focal chord:

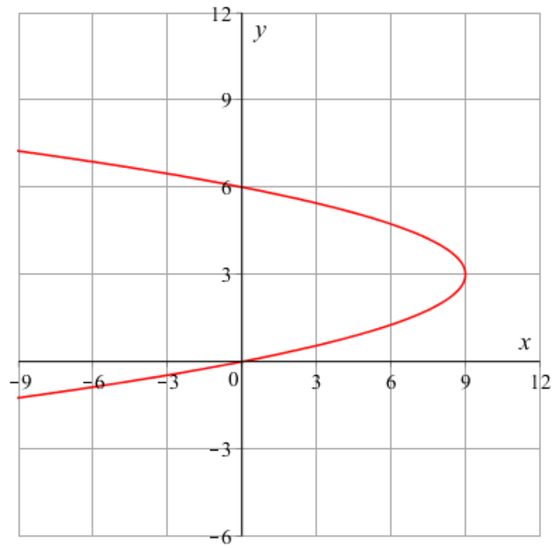


Example 5: Suppose you know that the vertex of a parabola is at (-3, 5) and its focus is at (1, 5). Write an equation for the parabola in standard form.

Example 6: Suppose you know that the focus of a parabola is (-1, 3) and the directrix is the line $y = -1$. Write an equation for the parabola in standard form.

Example 7. Find the equation of the parabola given by the following graph:

a.



b.