PRINTABLE VERSION

Practice Test 4

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

You scored 100 out of 100

Question 1

Your answer is CORRECT.

Which of the following is equivalent to

$$\frac{1 - cot^2(\theta)}{1 + cot^2(\theta)} + 2\,cos^2(\theta) = \frac{1 - \frac{\cos^2\theta}{\sin^2\theta}}{\csc^2\theta} + 2\cos^2\theta$$

$$= \frac{\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta}}{\left(\frac{1}{\sin^2\theta}\right)} + 2\cos^2\theta$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta} \cdot \frac{\sin^2\theta}{1} + 2\cos^2\theta = \sin^2\theta - \cos^2\theta + 2\cos^2\theta$$

$$= \sin^2\theta + \cos^2\theta = 1$$

a) $2\,sin^2(\theta)$

b) 1

c) 0

d) $-1 + 2\,cos^2(\theta)$

e) 2

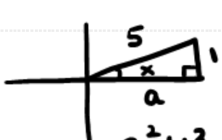
f) None of the above.

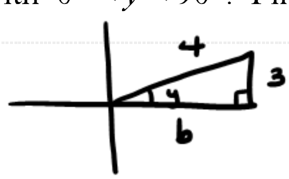
$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Question 2

Your answer is CORRECT.

Given $\sin(x) = 1/5$ with $0^o < x < 90^o$ and $\sin(y) = 3/4$ with $0^o < y < 90^o$. Find $\sin(x + y)$.





$a^2 + 1^2 = 5^2$

$a^2 = 24$

$a = 2\sqrt{6}$

$\cos(x) = \frac{2\sqrt{6}}{5}$

$b^2 + 3^2 = 4^2$

$b^2 = 16 - 9$

$b^2 = 7$

$b = \sqrt{7}$

$\cos(y) = \frac{\sqrt{7}}{4}$

a) $\frac{1}{10}\sqrt{2}$

b) $\frac{7}{20}$

c) $\frac{1}{20}\sqrt{7} - \frac{3}{10}\sqrt{6}$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{1}{5}\right)\left(\frac{\sqrt{7}}{4}\right) + \left(\frac{2\sqrt{6}}{5}\right)\left(\frac{3}{4}\right) = \frac{\sqrt{7}}{20} + \frac{6\sqrt{6}}{20}$

$= \frac{\sqrt{7}}{20} + \frac{3\sqrt{6}}{10}$

d) $\frac{1}{20}\sqrt{7} + \frac{3}{10}\sqrt{6}$

e) $\frac{79}{20}$

Question 3

$\sin(2x) = 2\sin(x)\cos(x)$

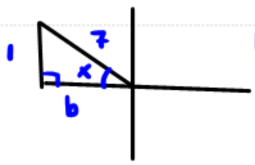
Your answer is CORRECT.

$\cos(x) = \frac{-1}{7}$

Given $\sec(x) = -7$ with $90° < x < 180°$. Find $\sin(2x)$.

$\sin(2x) = 2\left(\frac{4\sqrt{3}}{7}\right)\left(\frac{-1}{7}\right)$

$= \frac{-8\sqrt{3}}{49}$



$1^2 + b^2 = 7^2$

$b^2 = 48$

$b = 4\sqrt{3}$

a) $8\sqrt{3}/49$

b) $-8\sqrt{3}/49$

c) $-4\sqrt{3}/49$

d) $-10\sqrt{2}/49$

e) $-8\sqrt{3}/7$

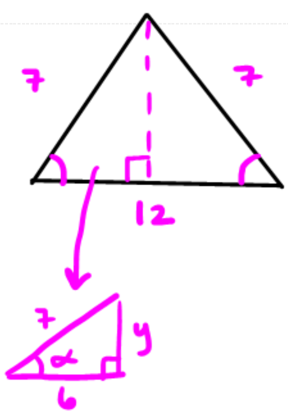
$\cos(x) = \frac{-1}{7}$

$\sin(x) = \frac{4\sqrt{3}}{7}$

Question 4

Your answer is CORRECT.

An isosceles triangle has sides 7, 7, and 12. Give the sine of a base angle of the triangle.



$\sin(\alpha) = \frac{\sqrt{13}}{7}$

a) $\frac{1}{6}\sqrt{13}$

b) $\frac{1}{7}\sqrt{13}$

c) $\frac{12}{7}$

d) $\frac{6}{13}\sqrt{13}$

e) $\frac{6}{7}$

$6^2 + y^2 = 7^2$

$y^2 = 49 - 36$

$y^2 = 13$

$y = \sqrt{13}$

Question 5

Your answer is CORRECT.

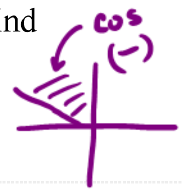
$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$$

Given

$$\cos(x) = -\frac{1}{4}$$

with $180^o < x < 270^o$. Find

$90 < \frac{x}{2} < 135$



$$\cos\left(\frac{1}{2}x\right) = -\sqrt{\frac{1+(-1/4)}{2}}$$

$$= -\sqrt{\frac{3/4}{2}}$$

$$= -\sqrt{\frac{3}{4}\cdot\frac{1}{2}} = -\sqrt{\frac{3}{8}}$$

$$= \frac{-\sqrt{3}}{2\sqrt{2}} = \frac{-\sqrt{6}}{4}$$

a) $-\frac{1}{2}\sqrt{3}$

b) $-\frac{1}{4}\sqrt{6}$

c) $-\frac{1}{4}\sqrt{10}$

d) $\frac{1}{2}\sqrt{6}$

e) $\frac{1}{4}\sqrt{6}$

Question 6

Your answer is CORRECT.

Give the number of solutions to the following equation on the interval $[0, 2\pi)$.

$$\sin^2(x) = \sin(x)$$

$\sin^2(x) - \sin(x) = 0$

$\sin(x)\,(\sin(x) - 1) = 0$

$\sin(x) = 0 \qquad \sin(x) = 1$

$x = 0, \pi \qquad x = \frac{\pi}{2}$

1 2 3

a) 1

b) 2

c) 3

d) 4

e) 0

Question 7

Your answer is CORRECT.

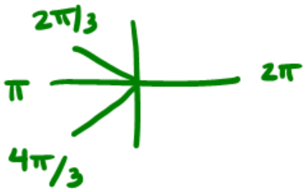
Find all solutions in the interval $[0\,,2\pi]$ of

$$2\cos^2(x) + 5\cos(x) + 2 = 0$$

$$(2\cos(x) + 1)(\cos(x) + 2) = 0$$

$$2\cos(x) = -1 \qquad \cos(x) = -2 \notin [-1,1]$$

$$\cos(x) = -\frac{1}{2}$$



a) $\{\, 7\pi/6\,,\ 11\pi/6 \,\}$

b) $\{\, \pi/6\,,\ \pi\,,\ 11\pi/6\,,\ 3\pi/2 \,\}$

c) $\{\, \pi/2\,,\ \pi \,\}$

d) $\{\, 2\pi/3\,,\ 4\pi/3 \,\}$

e) $\{\, \pi/3\,,\ \pi/4 \,\}$

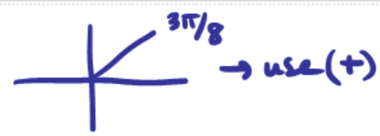
Question 8

Your answer is CORRECT.

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$

Evaluate:

$$\sin\left(\frac{3}{8}\pi\right)$$



$$\sin\left(\frac{3\pi/4}{2}\right) = \sqrt{\frac{1-\cos(3\pi/4)}{2}}$$

$$= \sqrt{\frac{1+\sqrt{2}/2}{2}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$

b) $-\frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}$

c) $\frac{1}{2}\sqrt{2+\sqrt{2}}$

d) $-\frac{1}{2}\sqrt{2-\sqrt{2}}$

e) $-\frac{1}{2}\sqrt{2+\sqrt{2}}$

Question 9

Your answer is CORRECT.

Find $\sin(105^{o})$ using the sum or difference formulas.

a) ○ $\frac{1}{2}\sqrt{3}$

b) ◉ $\frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6}$

c) ○ $\frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{3}$

d) ○ $\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6}$

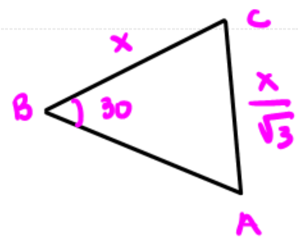
e) ○ $\frac{1}{2}\sqrt{2}$

$$\sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

Question 10

Your answer is CORRECT.

The following facts are true about triangle ABC, where the measure of angle B is 30°, BC = x, and AC = $x/\sqrt{3}$. How many choices are there for the measure of angle A?



SSA

$$\frac{\sin A}{x} = \frac{\sin 30}{\frac{x}{\sqrt{3}}}$$

$$\frac{x}{\sqrt{3}} \sin A = \frac{x}{2}$$

$$\sin A = \frac{x}{2}\cdot\frac{\sqrt{3}}{x} = \frac{\sqrt{3}}{2}$$

A = 60

A = 120

check

$60 + 30 < 180$ ✓

$120 + 30 < 180$ ✓

a) ○ 4

b) ○ 3

c) ○ 1

d) ◉ 2

e) ○ 0

Question 11

$\sin^2\theta = 1 - \cos^2\theta$

Your answer is CORRECT.

Simplify the following expression:

$$\frac{\sin^2(\theta)}{1-\cos(\theta)} - 1 = \frac{(1-\cos^2\theta)}{(1-\cos\theta)} - 1$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)} - 1$$

$$= 1 + \cos\theta - 1 = \cos\theta$$

a) ○ $\frac{\cos(\theta)+1}{\cos(\theta)-1}$

b) ○ $\sec(\theta)$

c) $\tan(\theta)$

d) $\sin(\theta)$

e) $\cos(\theta)$

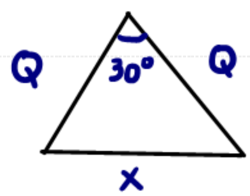
Question 12

Your answer is CORRECT.

$a^2 = b^2 + c^2 - 2bc \cos A$

An isosceles triangle has an apex angle measuring 30^o. The two equal sides have length Q inches. What is the length of the base (in inches)?

SAS



a) $\sqrt{2Q^2 - 2Q\cos(30^o)}$

b) $\sqrt{2Q^2 - Q^2\cos(30^o)}$

c) $\sqrt{2Q^2 - 2Q^2\cos(30^o)}$

d) $\sqrt{2Q^2 + 2Q^2\cos(30^o)}$

e) $\sqrt{2Q^2 + Q\cos(30^o)}$

$x^2 = Q^2 + Q^2 - 2QQ\cos 30$

$x^2 = 2Q^2 - 2Q^2 \cos 30$

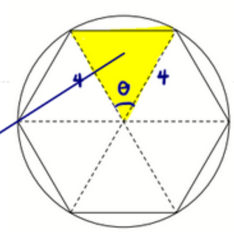
$x = \sqrt{2Q^2 - 2Q^2 \cos 30}$

Question 13

$A_{HEX} = 6(A_\Delta)$

Your answer is CORRECT.

Find the area of a regular hexagon inscribed in a circle of radius 4 cm.



$A_\Delta = \frac{1}{2} ab \sin\theta$

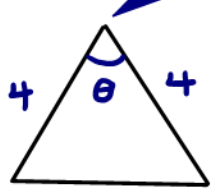
a) $32\sqrt{2}$ cm^2

$\frac{360}{6} = 60$

b) $48\sqrt{3}$ cm^2

$A_{HEX} = 6(4\sqrt{3})$

$= \boxed{24\sqrt{3}}$



$A_\Delta = \frac{1}{2}(4)(4)\sin 60$

$A_\Delta = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$

c) $12\sqrt{3}$ cm ^{2}

d) $24\sqrt{3}$ cm^2

e) 32 cm^2

Question 14

Your answer is CORRECT.

$\sin(\theta) = 1 \longrightarrow [0, 2\pi]$

$\theta = \frac{\pi}{2}$

Solve

$$\sin\left(\pi x + \left(\frac{1}{7}\pi\right)\right) = 1$$

over the interval $[\,^{-1}\!/_{7}\,, ^{6}\!/_{7}\,]$

$\downarrow$

$\left[\frac{-2}{14}, \frac{12}{14}\right]$

$\pi x + \frac{\pi}{7} = \frac{\pi}{2} + 2\pi k$

$-\frac{\pi}{7} \quad -\frac{\pi}{7}$

$\pi x = \frac{7\pi}{14} - \frac{2\pi}{14} + 2\pi k$

$\frac{\pi x}{\pi} = \frac{\frac{5\pi}{14} + 2\pi k}{\pi}$

$x = \frac{5}{14} + 2k = \frac{5}{14} + \frac{28}{14}k$

$k=0$

$\boxed{x = \frac{5}{14}}$

a) $\frac{9}{14}$

b) $\frac{1}{14}\pi$

c) $\frac{3}{7}$

d) $\frac{1}{4}\pi + \frac{7}{4}$

e) $\frac{5}{14}$

Question 15

Your answer is CORRECT.

A string running from the ground to the top of a fence has an angle of elevation of 45°. The string is 10 feet long. What is the distance between the fence and where the string is pegged to the ground?

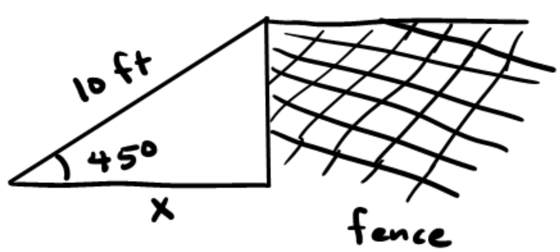
a) 5 feet

b) 20 feet

c) $\sqrt{10}$ feet

d) $5\sqrt{2}$ feet

e) $5\sqrt{3}$ feet



$\cos(45) = \frac{x}{10}$

$\frac{\sqrt{2}}{2} = \frac{x}{10}$

$2x = 10\sqrt{2}$

$x = 5\sqrt{2}$ ft

Question 16

Your answer is CORRECT.

In triangle ABC,

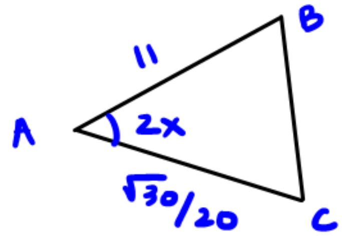
The measure of angle A is $2x$,
$AB = 11$, and

$\sin 2x = 2\sin x \cos x$

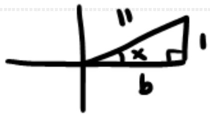
$\sin 2x = 2\left(\frac{1}{11}\right)\left(\frac{2\sqrt{30}}{11}\right)$

$= 4\sqrt{30}/121$

$$AC = \frac{1}{20}\sqrt{30}$$



If you know that $\sin(x) = 1/11$, what is the area of the triangle?



$b^2 + 1^2 = 11^2$

$b^2 = 121 - 1$

$b^2 = 120$

$b^2 = 2\sqrt{30}$

$\cos(x) = \frac{2\sqrt{30}}{11}$

$A = \frac{1}{2} bc \sin 2x$

$A = \frac{1}{2}\left(\frac{\sqrt{30}}{20}\right)(11)\left(\frac{4\sqrt{30}}{121}\right)$

$A = \frac{30(4)}{20 \cdot 2} = \frac{12}{4 \cdot 11} = \frac{6}{22} = \frac{3}{11}$

a) 120

b) 11

c) 22

d) $3/11$

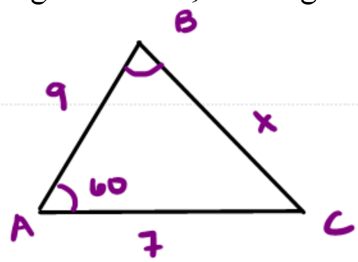
e) $\sqrt{2}/2$

Question 17

Your answer is CORRECT.

Given triangle ABC with the measure of angle A = 60°, the length of AB = 9, and the length of AC = 7. What is the length of side BC?

SAS



a) $\sqrt{107 - \sqrt{2}}$

b) 58

c) $\sqrt{67}$

d) $\sqrt{130}$

e) $\frac{9}{2}$

$x^2 = 9^2 + 7^2 - 2(9)(7)\cos 60$

$x^2 = 81 + 49 - 126\left(\frac{1}{2}\right)$

$x^2 = 81 + 49 - 63 = 67$

$x^2 = 67$

$x = \sqrt{67}$

Question 18

Your answer is CORRECT.

Determine all solutions to

$$\sin(2x) = \frac{1}{2}$$

on the interval $[0, 2\pi)$.

$\sin(\theta) = \frac{1}{2} \rightarrow [0, 2\pi]$

$\theta = \pi/6 \quad \theta = 5\pi/6$

$2x = \frac{\pi}{6} + 2\pi k \qquad 2x = \frac{5\pi}{6} + 2\pi k$

$x = \frac{\pi}{12} + \pi k \qquad x = \frac{5\pi}{12} + \pi k$

$x = \frac{\pi}{12} + \frac{12\pi}{12} k \qquad x = \frac{5\pi}{12} + \frac{12\pi}{12} k$

a) $\left\{\frac{1}{3}\pi, \frac{2}{3}\pi\right\}$

b) $\left\{\pi, \frac{1}{3}\pi, \frac{1}{6}\pi, \frac{5}{6}\pi\right\}$

c) $\left\{\frac{1}{12}\pi, \frac{5}{12}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi\right\}$

d) $\left\{\frac{1}{3}\pi, \frac{1}{6}\pi, \frac{4}{3}\pi, \frac{7}{6}\pi\right\}$

e) $\left\{\frac{1}{6}\pi, \frac{5}{6}\pi\right\}$

$k=0 \quad x_1 = \boxed{\frac{\pi}{12}} \quad x_2 = \boxed{\frac{5\pi}{12}}$

$k=1 \quad x_1 = \boxed{\frac{13\pi}{12}} \quad x_2 = \boxed{\frac{17\pi}{12}}$

$k=2 \quad x_1 = \frac{25\pi}{12} \times \quad x_2 = \frac{29\pi}{12} \times$

$[0, 2\pi) \downarrow \left[\frac{0}{12}, \frac{24\pi}{12}\right)$

Question 19

Your answer is CORRECT.

Draw triangle ABC with side BC as the base. The measure of angle A = 45°. If a perpendicular is drawn from B to side AC, the height is 8. What is the length of side AB?

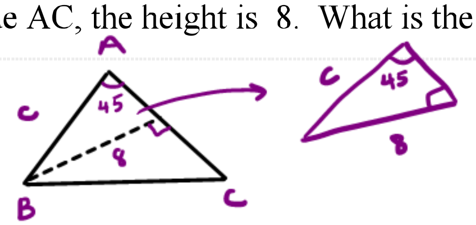
a) $8\sqrt{3}$

b) $8\sqrt{2}$

c) $4\sqrt{2}$

d) $4\sqrt{6}$

e) 16



$\sin 45 = \frac{8}{c}$

$\frac{\sqrt{2}}{2} = \frac{8}{c}$

$c(\sqrt{2}) = 16$

$c = \frac{16}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$

$a\sqrt{2} = c$

$c = 8\sqrt{2}$

Question 20

Your answer is CORRECT.

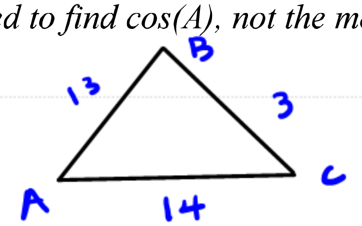
ABC is a triangle with AB = 13, BC = 3, and AC = 14. Find cos(A).
Note: You are asked to find cos(A), not the measure of angle A. Do not use a calculator.

a) 9/91

b) 89/91

c) 14/13

d) 3/13



$a^2 = b^2 + c^2 - 2bc \cos A$

$3^2 = 14^2 + 13^2 - 2(14)(13)\cos A$

$9 = 196 + 169 - 364 \cos A$

$-356 = -364 \cos A$

$\cos A = \frac{356}{364} = \frac{89}{91}$

e) $^{178}/_{91}$

Question 21

Your answer is CORRECT.

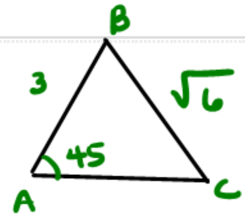
SSA

Given triangle ABC with

$$AB = 3$$

and

$$BC = \sqrt{6}$$



The measure of angle A is 45^o. How many choices are there for the measure of angle C?

a) 0

b) 4

c) 2

d) 1

e) 3

$C = 60^\circ$

$C = 120^\circ$

Check

$120 + 45 < 180$ ✓

$60 + 45 < 180$ ✓

$\frac{\sin 45}{\sqrt{6}} = \frac{\sin C}{3}$

$3\left(\frac{\sqrt{2}}{2}\right) = \sqrt{6} \sin C$

$\frac{3\sqrt{2}}{2} = \sqrt{6} \sin C$

$\frac{3\sqrt{2}}{2\sqrt{6}} = \sin C$

$\sin C = \frac{3\sqrt{12}}{12}$

$\sin C = \frac{2\sqrt{3}}{6}$

$\sin C = \frac{\sqrt{3}}{2}$

Question 22

Your answer is CORRECT.

Solve

$$3 \sin(x) \cos^2(x) = 3 \sin(x)$$

on the interval $[-\pi, \pi]$.

$3\sin(x)\left(\cos^2(x) - 1\right) = 0$

a) $\left\{0, \pi, -\pi, -\frac{1}{4}\pi, \frac{1}{4}\pi\right\}$

b) $\left\{\frac{1}{4}\pi, \frac{3}{4}\pi\right\}$

c) $\left\{0, -\frac{1}{2}\pi, \frac{1}{2}\pi\right\}$

d) *No Solution*

e) $\{0, \pi, -\pi\}$

$3\sin(x) = 0$

$\sin x = 0$

$x = -\pi$

$x = 0$

$x = \pi$

$\cos^2(x) - 1 = 0$

$(\cos(x) + 1)(\cos(x) - 1) = 0$

$\cos x = -1$ $\quad$ $\cos x = 1$

$x = \pi$ $\quad$ $x = 0$

Question 23

Your answer is CORRECT.

ABC is a triangle with angle A = 60^{o}, angle B = 45^{o}, and BC = 7. Find AC.

a) $7\sqrt{6}/2$

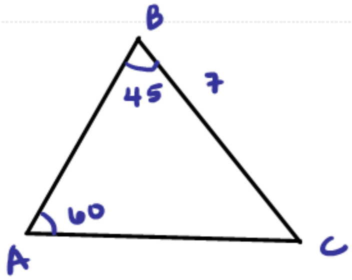
b) $7\sqrt{6}/3$

c) 49

d) $21/2$

e) $14/3$

f) None of the above.



AAS

$$\frac{\sin 60}{7} = \frac{\sin 45}{b}$$

$$\frac{\sqrt{3}}{2(7)} = \frac{\sqrt{2}}{2b}$$

$$2\sqrt{3}b = 14\sqrt{2}$$

$$b = \frac{14\sqrt{2}}{2\sqrt{3}} = \frac{14\sqrt{6}}{6} = \frac{7\sqrt{6}}{3}$$

Question 24

Your answer is CORRECT.

Two cyclists leave the corner of State Street and Main Street simultaneously. State Street and Main Street are not at right angles; the cyclists' paths have an angle of 45^{o} between them. How far apart are the cyclists after they each travel 5 miles? *Hint: Use the Law of Cosines*

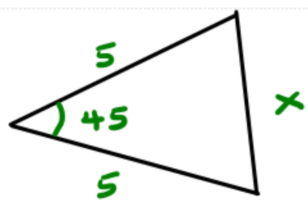
a) $10 - \sqrt{5}$

b) 10

c) $\sqrt{25 - \sqrt{5}}$

d) 5

e) $5\sqrt{2 - \sqrt{2}}$



$$x^2 = 5^2 + 5^2 - 2(25)\cos 45$$

$$x^2 = 50 - 50\frac{\sqrt{2}}{2}$$

$$x^2 = 50 - 25\sqrt{2}$$

$$x = \sqrt{50 - 25\sqrt{2}}$$

$$x = \sqrt{25(2 - \sqrt{2})}$$

$$x = 5\sqrt{2 - \sqrt{2}}$$

Question 25

Your answer is CORRECT.

Given $\sin(x) = 2/3$ with $0^{o} < x < 90^{o}$, and $\sin(y) = -7/9$ with $180^{o} < y < 270^{o}$. Find $\cos(x - y)$.

a) $\dfrac{-4\sqrt{10} - 14}{(27)}$

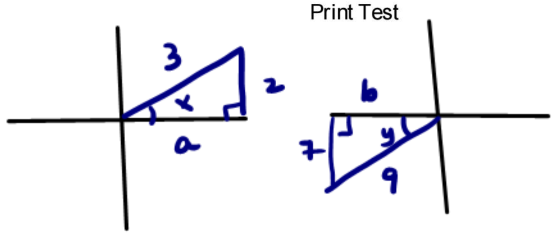
b) $\frac{2\sqrt{2}+7}{(27)}$

c) $-\frac{12}{(27)}$

d) $\frac{160}{27}$

e) $\frac{4\sqrt{10}}{(27)}$

f) *None of the above.*



$7^2+b^2=9^2$

$b^2=81-49$

$b^2=32$

$b=4\sqrt{2}$

$a^2+2^2=3^2$

$a^2=9-4$

$a=\sqrt{5}$

$\cos(x-y)=\cos x\cos y+\sin x\sin y$

$=\left(\frac{\sqrt{5}}{3}\right)\left(\frac{-4\sqrt{2}}{9}\right)+\left(\frac{2}{3}\right)\left(\frac{-7}{9}\right)$

$=-\frac{4\sqrt{10}}{27}-\frac{14}{27}=\frac{-4\sqrt{10}-14}{27}$