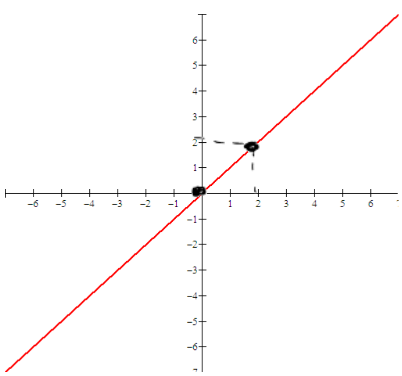
Practice these basic functions and their graphs. Whenever asked, you should be able to visualize the shape of each.

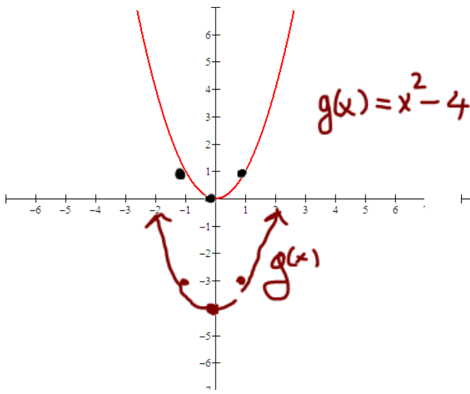
Math 1330 Section 1.3
Transformations of Graphs

In College Algebra, you should have learned to transform nine basic functions. Here are the basic functions. You should know the shapes of each graph, domain and range of the function, and you should be able to state intervals on which the function is increasing and intervals on which the function is decreasing.

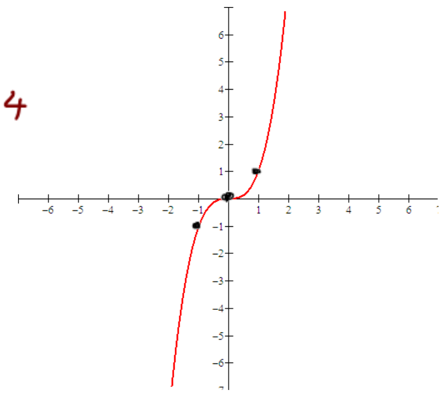
$f(x) = x$ (need two points)



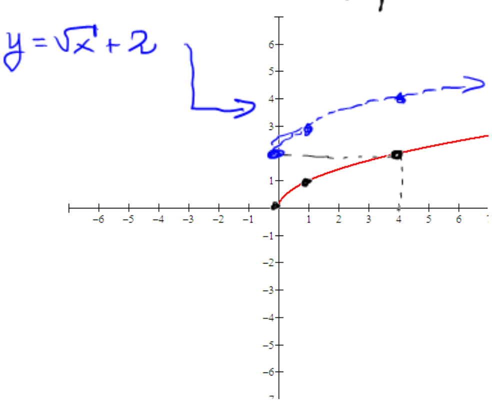
$f(x) = x^2$ (at least 3 points)



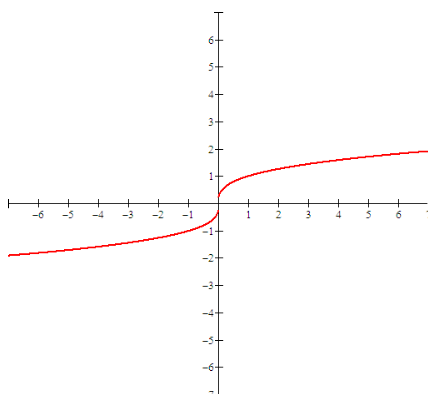
$f(x) = x^3$ (at least 3 points)



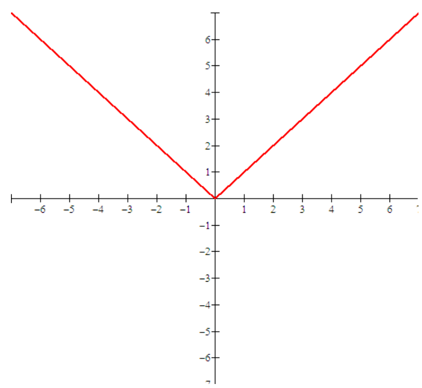
$f(x) = \sqrt{x}$ (3 points)



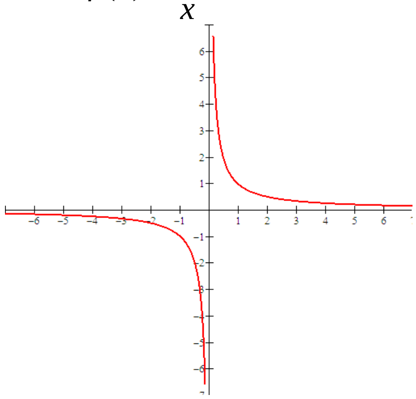
$f(x) = \sqrt[3]{x}$



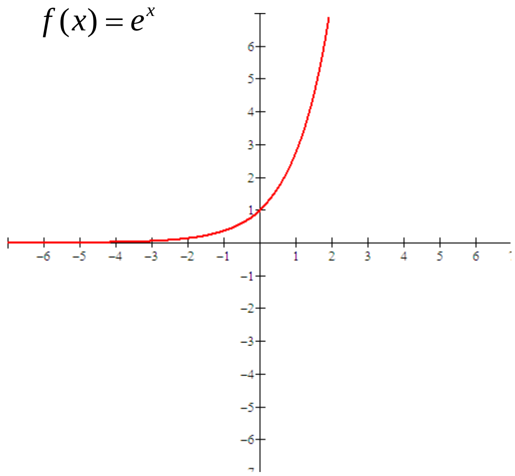
$f(x) = |x|$



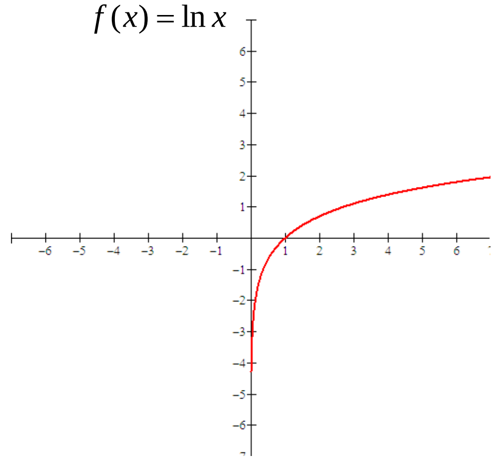
$f(x) = \frac{1}{x}$



$f(x) = e^x$



$f(x) = \ln x$



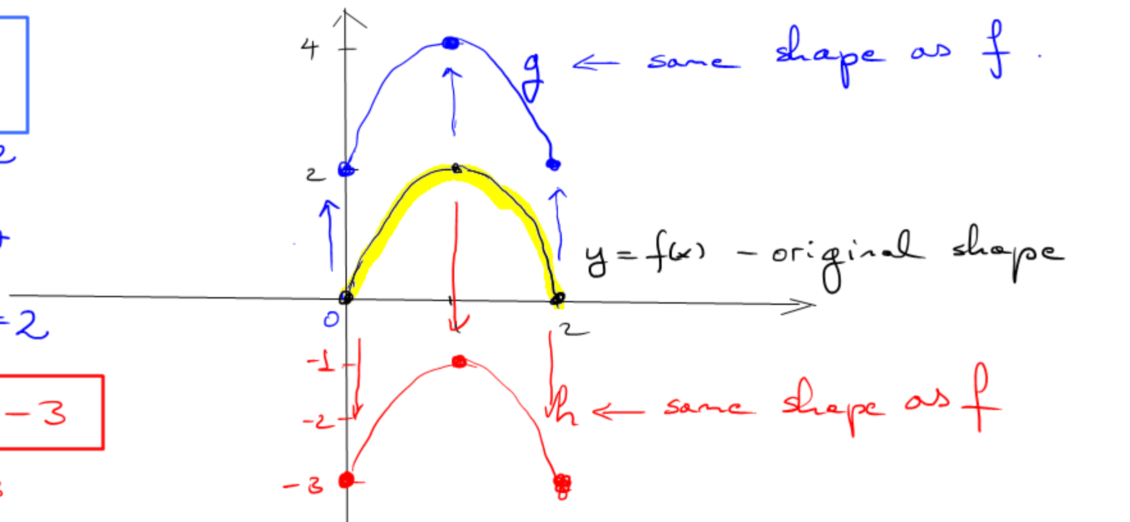
You should be able to translate these graphs vertically and/or horizontally, reflect them about the x or the y axis, and stretch them or shrink them vertically or horizontally.

Vertical Shifting $y = f(x) \leftarrow$ shape of the graph

To graph $f(x)+c$, $(c>0)$, start with the graph of $f(x)$ and shift it upward c units.

To graph $f(x)-c$, $(c>0)$, start with the graph of $f(x)$ and shift it downward c units.

- graph $\boxed{g(x)=f(x)+2}$
  - $x=0$, $g(0)=f(0)+2=2$
  - $x=1$, $g(1)=f(1)+2=4$
  - $x=2$, $g(2)=f(2)+2=2$
- graph $\boxed{h(x)=f(x)-3}$
  - $h(0)=f(0)-3=-3$
  - $h(1)=f(1)-3=-1$
  - $h(3)=f(2)-3=-3$

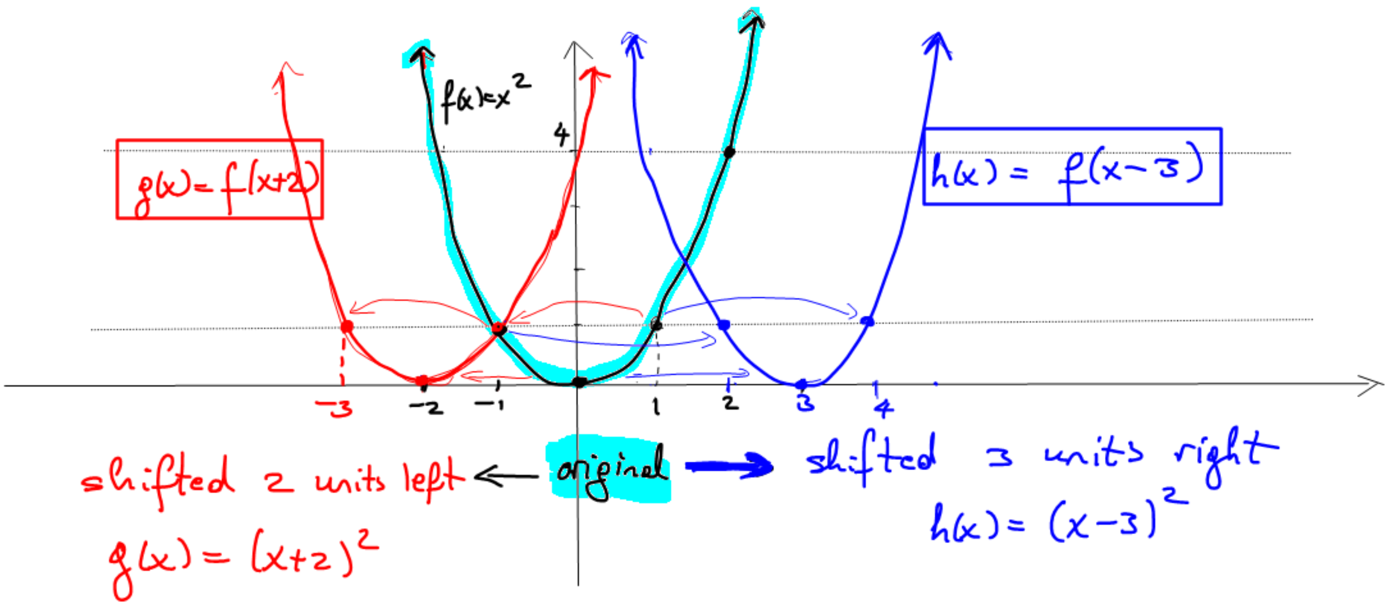


Note: x-values do NOT change.

Horizontal Shifting

To graph $f(x+c)$, $(c>0)$ start with the graph of $f(x)$ and shift it to the left c units.

To graph $f(x-c)$, $(c>0)$, start with the graph of $f(x)$ and shift it to the right c units.



Note: y-values do NOT change!

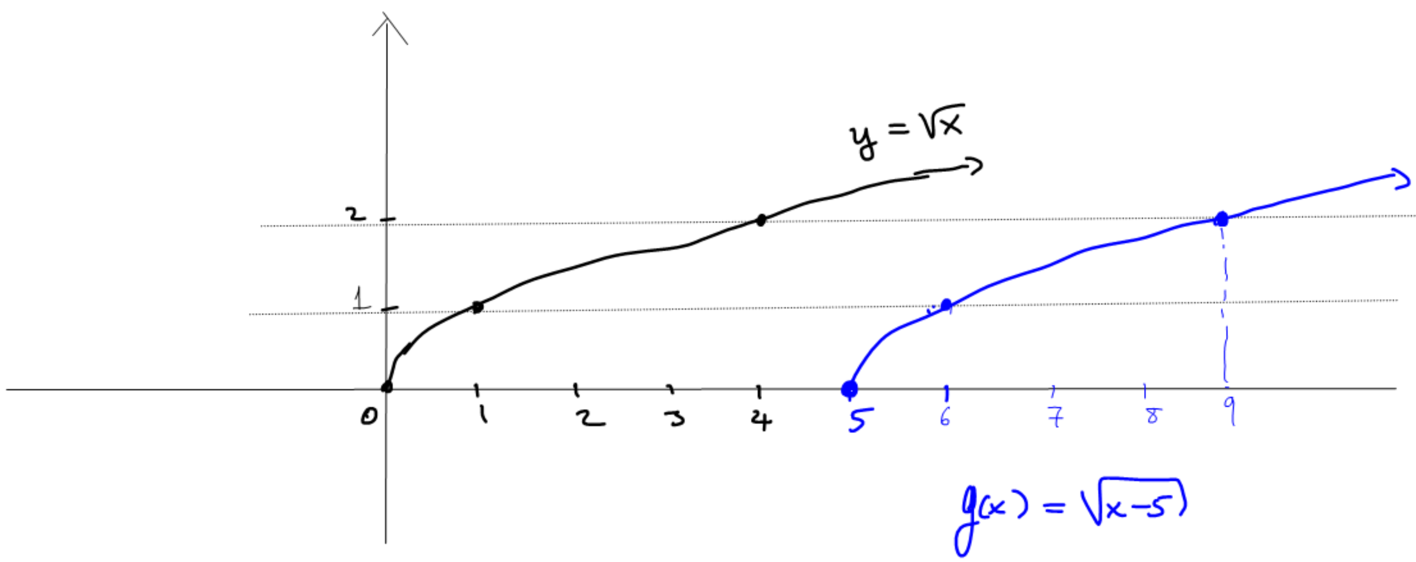
example:

Graph $g(x) = \sqrt{x-5}$.

Let's break it down:

- shape of graph : $y=\sqrt{x}$ ← parent fn.

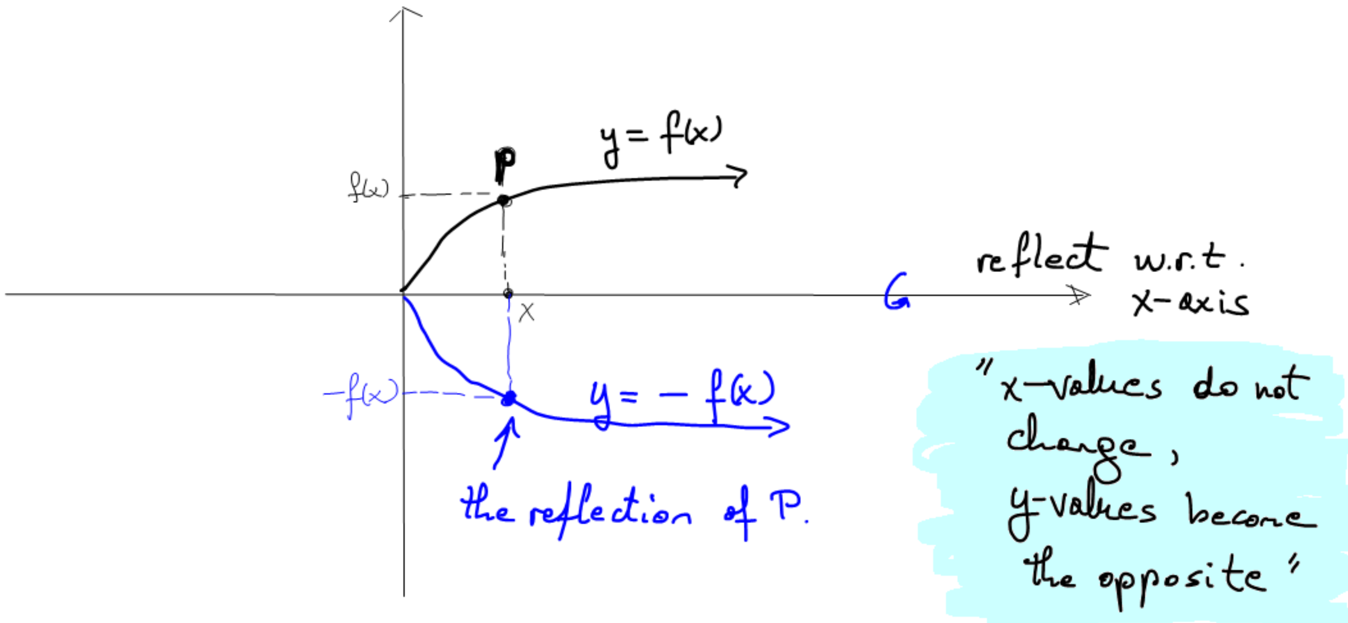
- transformation : $\underline{x-5}$ $\underbrace{\text{shift 5 units right}}$



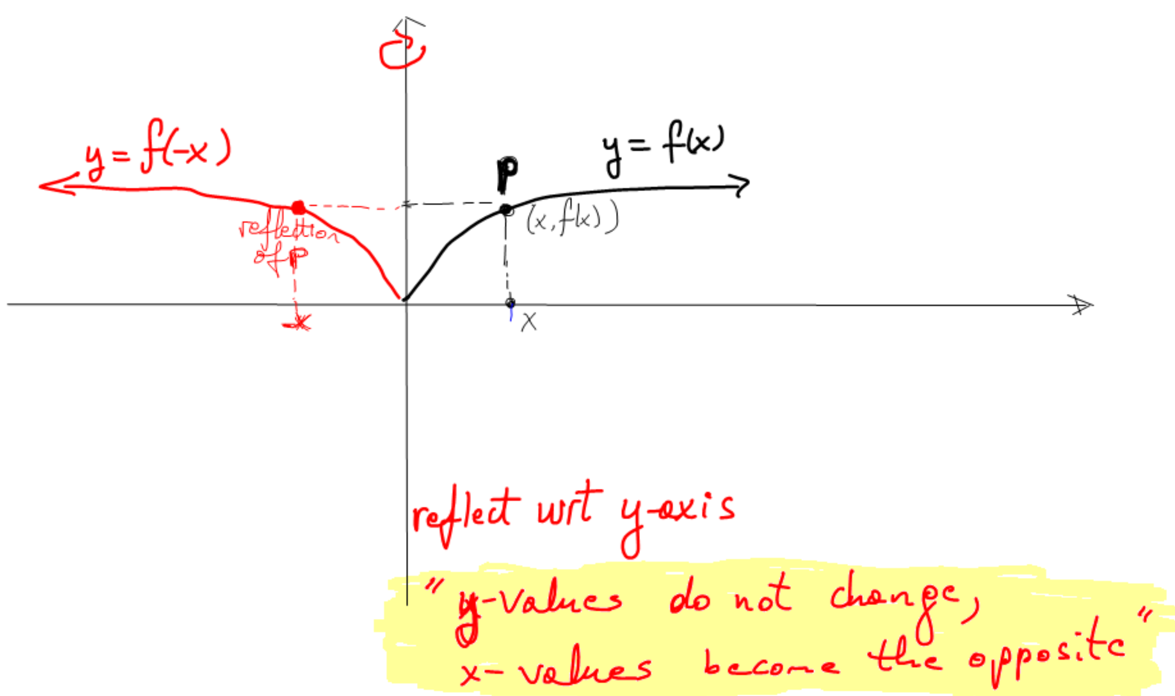
Reflection of Functions

A reflection is the "mirror-image" of graph about the x-axis or y-axis.

To graph $y = -f(x)$, reflect the graph of $f(x)$ about the x-axis.



To graph $y = f(-x)$, reflect the graph of $f(x)$ about the y-axis.

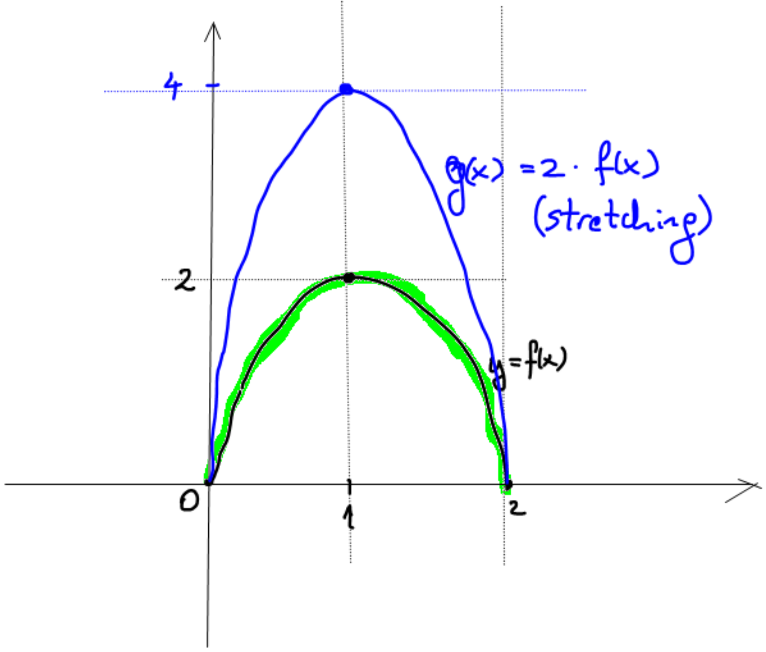


$0<a<1$

Vertical Stretching and Shrinking $y = a \cdot f(x)$

$a>1$

Vertical Stretching: If $\boxed{a > 1}$ the graph of $y = af(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of its y -coordinates by a .



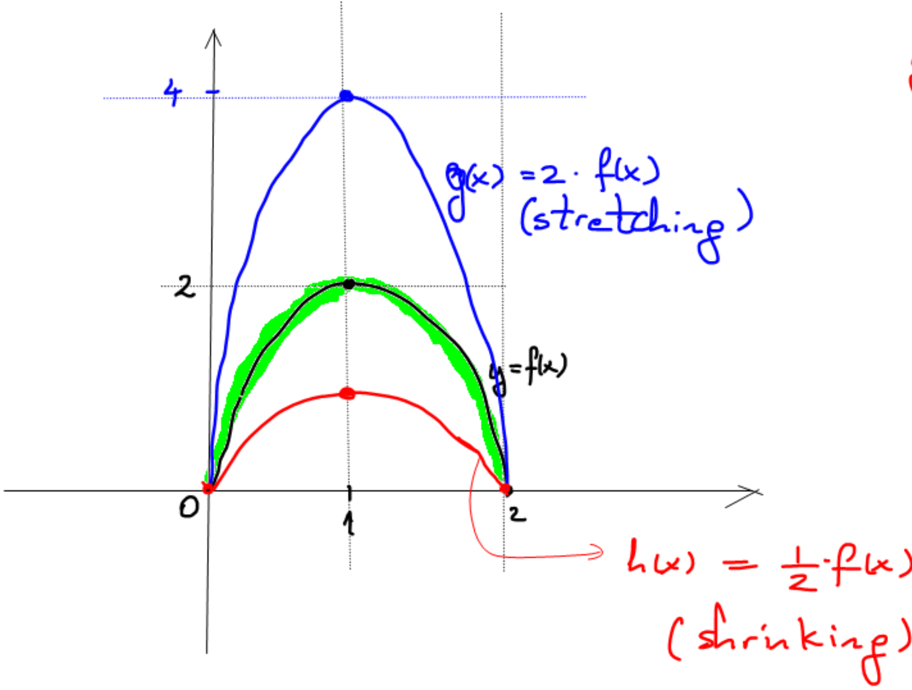
Graph $g(x) = 2 \cdot f(x)$

$x=0$, $g(0) = 2 \cdot f(0) = 0$

$x=1$, $g(1) = 2 \cdot f(1) = 4$

$x=2$, $g(2) = 2 \cdot f(2) = 0$

Vertical Shrinking: If $\boxed{0< a < 1,}$ the graph of $y = af(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of its y-coordinates by a.



graph $h(x) = \frac{1}{2} \cdot f(x)$

$x=0$, $h(0) = \frac{1}{2} \cdot f(0) = 0$

$x=1$, $h(1) = \frac{1}{2} \cdot f(1) = 1$

$x=2$, $h(2) = \frac{1}{2} \cdot f(2) = 0$

You may find it helpful to apply transformations in this order:

1. **Vertical and/or Horizontal Stretching or Shrinking**
2. **Reflection about the *x* axis**
3. **Horizontal or Vertical translations**
4. **Reflection about the *y* axis**

This order will help you to get the right answer, in case you are not sure.

This is not the only order which works, but you will make few mistakes if you apply transformations in this order.

Example 1: Suppose you are asked to graph the function $f(x) = -\sqrt{x+2} - 5$. Start with the function $g(x) = \sqrt{x}$ and state the transformations needed and the order in which you would apply them in order to sketch $f(x)$.

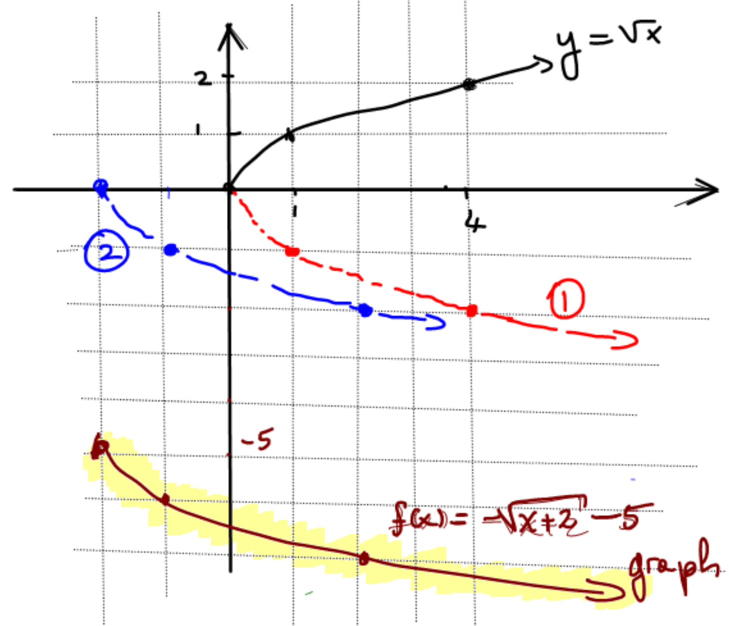
$f(x) = -\sqrt{x+2} \;\; - 5$

Start with $y = \sqrt{x}$ ← shape

① reflect wrt. x-axis

② shift 2 units left

③ shift 5 units down



Example 2: Suppose you are asked to graph the function $g(x) = -5f(-x) + 4$. Starting with the graph of $f(x)$, state the transformations needed and the order in which you would apply them in order to sketch $g(x) = -5f(-x) + 4$.

• Start with $y = f(x)$ ← parent

① vertical stretch by factor of 5: $y \to 5 \cdot y$ $(x, y) \to (x, 5y)$

② reflect wrt x-axis: $y \to -y$ $(x, y) \to (x, -y)$

③ reflect wrt y-axis: $x \to -x$ $(x, y) \to (-x, y)$

④ shift 4 units up: $y \to y+4$ $(x, y) \to (x, y+4)$

If the point (2,1) is on the graph of $f(x)$, which point should be on the graph of $g(x) = -5f(-x) + 4$. ?

original (x, y): $(2, 1) \xrightarrow{①} (2, 5) \xrightarrow{②} (2, -5) \xrightarrow{③} (-2, -5) \xrightarrow{④} (-2, -1)$

$5 \cdot 1 = 5$ -5 -2 $y+4 = -5+4 = -1$

$\Rightarrow$ $\boxed{(2,1) \longrightarrow (-2, -1)}$

exercise

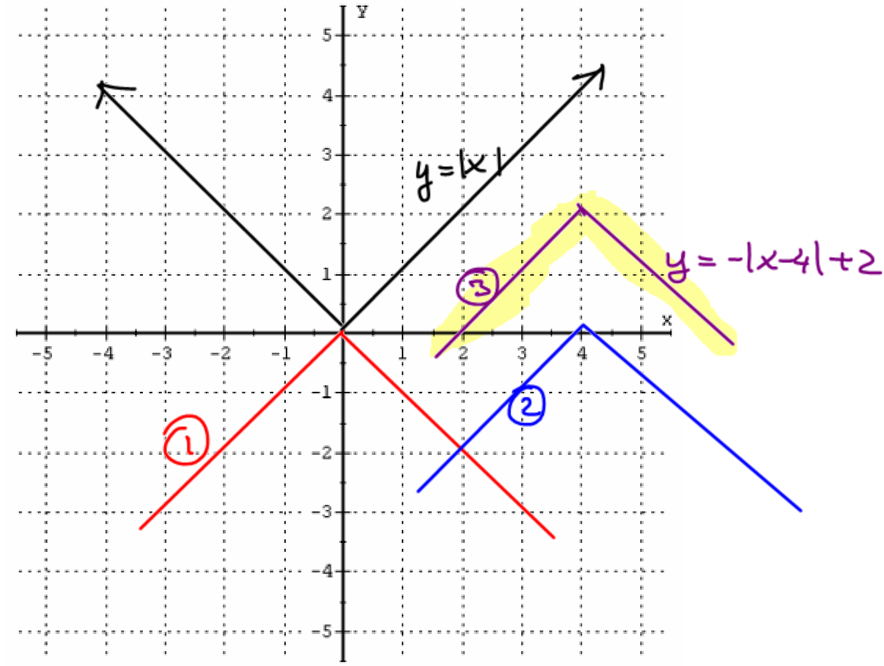
Example 3: Sketch $f(x) = -|x-4| + 2$ using transformations.

Start with $y = |x|$.

↳ ① reflect wrt x-axis

↳ ② shift 4 units right

↳ ③ shift 2 units up

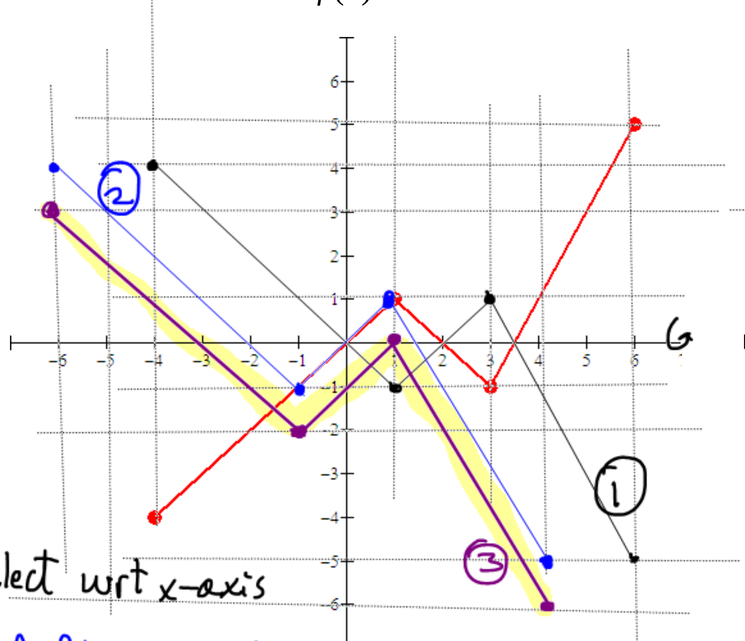


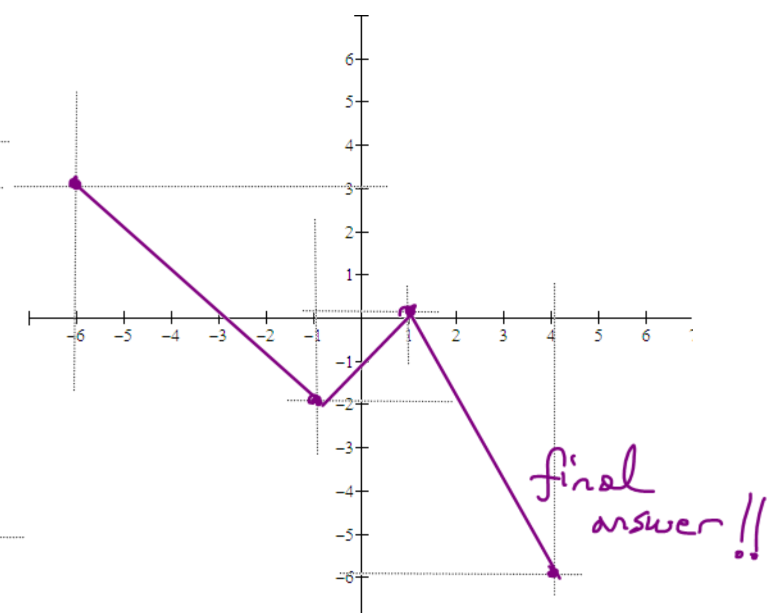
Sometimes, you'll be given a graph with no statement of the function and you'll need to be able to graph a transformed version of it. In this case, it will be helpful to look at key points on the graph of the function, transform those and graph them as a guide to graphing the transformed version. Here is an example:

exercise

Example 4: Suppose you are given the graph of a function, $f(x)$. Use it to sketch . $-f(x+2)-1$.

$f(x)$





① reflect wrt x-axis

② shift 2 left ,

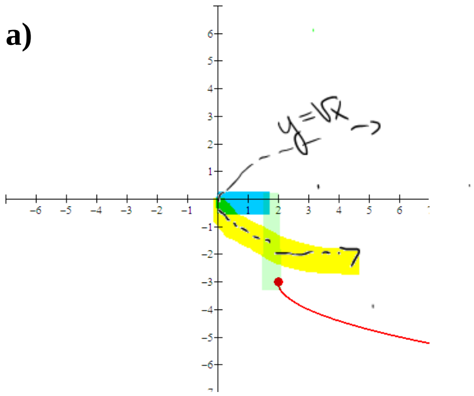
③ shift 1 unit down .

exercise.

Now suppose you are given the graph of a function and you are asked to write the function. You'll need to be able to identify the basic function that's given and then describe all of the transformations that were applied to it. From that, you should be able to write the function.

Example 5: Write the function that is graphed here.

a)



It looks like $y=\sqrt{x}$

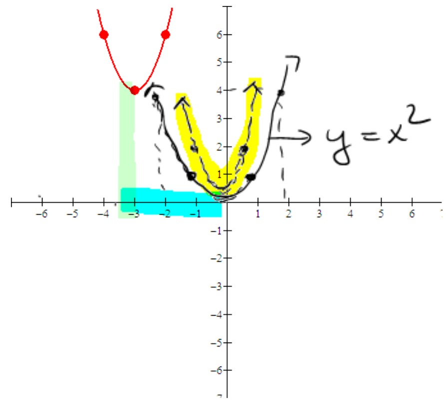
reflected wrt x-axis $\to$ $y=-\sqrt{x}$

shifted 2 units right $\to$ $y=-\sqrt{x-2}$

shifted 3 units down $\to$ $y=-\sqrt{x-2}-3$

$$\Rightarrow \boxed{y=-\sqrt{x-2}-3}$$

b)



It looks like $y=x^2$ but stretched vertically by a factor of 2

$\to$ $y=2x^2$

shifted 3 to left $\to$ $y=2(x+3)^2$

shifted 4 up $\to$ $y=2(x+3)^2+4$

$$\boxed{y=2(x+3)^2+4}$$